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Date April 20, 1978

ABSTRACT

The contents of this thesis belongs entirely to the area of instrumental acoustics. It is written in the style of a textbook for music students, and aims to serve an instructional and informative purpose. The first goal of the paper is to outline the manner in which woodwind and brass instruments function. Similarities and differences between individual instruments and families of instruments are discussed.

The basis of the thesis is a disclosure of the physical properties of sound waves in general, waves in cylindrical tubes, waves in conical tubes, and waves in both these types of tubes with certain boundary conditions imposed on them. These properties are directly applied to the woodwind instruments, but they cannot be applied to brass instruments; consequently, a different procedure is adopted for their discussion.

The original goal of the author was to discuss the most salient known, proven mathematical and acoustical properties of woodwind and brass instruments. In the process of doing this, conclusions

evolved which led to a claim and subsequent formation of theories which are, to the best of the writer's knowledge, presented here for the first time, and which still remain to be completely proven by complex mathematical methods.

TABLE OF CONTENTS

LIST OF TABLES FOR TESTING THE EQUATION $v = f\lambda$

LIST OF TABLES OF FREQUENCIES

LIST OF ILLUSTRATIONS

PREFACE

Acoustics in general and the acoustics of instruments in particular are aspects of music, mathematics, and physics which continue to be generally neglected and unlearned by most students and professors concerned with these disciplines. In general, the root of this problem lies in the fact that mathematicians and physicists are most often primarily interested in aspects of their fields other than music, while musicians are interested in almost any aspect of music other than the mathematical and physical. From the point of view of a musician and mathematician (but not that of a physicist!), the writer sees this as most unfortunate. The three areas are so interrelated and enhance each other so greatly that it is a wonder that so few cultures have grouped them into one discipline, and relatively few people bother to study these disciplines in relation to each other.

From the point of view of a music theorist, the writer candidly admits that we as musicians have more to gain from studying mathematical and physical aspects of music than our counterparts in those two disciplines have to gain by studying musical aspects of mathematics and physics. (It follows, therefore, that the converse is not true.) Why, then, are musicians (some of whom spend all their lives studying music) so reluctant to study sound?

One common answer to this question is that most musicians are understandably interested in instrumental (or vocal) performance, and not in theoretical study. This, however, is perhaps a weak excuse for ignorance about how instruments operate and why they function the way they do. In the most pitiful cases, musicians don't even understand the fundamental properties of their own instrument. The area of musical acoustics is the one which best provides this information to musicians; this paper is directed towards that goal.

The first purpose of this work is to present a study of the principles of operation of woodwind and brass instruments. Since the object of the undertaking is to outline these properties in an informative manner, (at an elementary level only), the textbook style of writing is employed throughout. For each instrument which is discussed, the reader may expect to find, in this thesis, a brief answer to the following three questions: A) How does this instrument operate? B) Why does this instrument function the way it does? C) In what basic ways does the operation of this instrument compare and contrast with the operation of the other eight instruments which are discussed? The best example of how these questions are answered is provided by the case of the clarinet. In this case, the explanation of fingering and overblowing becomes equivalent to the explanation of the contrast between the clarinet and all the other woodwind instruments.

It is impossible to answer any of the above questions without first examining the physical properties which govern the behavior of sound. This is why Section I does not deal with instruments, but with sound waves. For the same reason, Sections II and III begin with a discussion of tube shapes and the behavior of sound waves under specific boundary conditions.

Since this paper is directed primarily at musicians (and not at physicists or mathematicians), the amount of physics and mathematics has been reduced to a minimum. Section I, together with the introductory portions of Sections II and III, represents this minimum, containing about 90% of all the mathematics in the paper. Consequently, these sections do not reflect the nature of the content of the remainder of the thesis.

The second purpose of this work was not originally anticipated, but came about only during the course of preparation of Section IV. With a glance at the Table of Contents, the reader will notice that "Testing the Equation $v = f\lambda$" permeates the entire body of the work. For the woodwind instruments, this experiment was carried out purely to satisfy curiosity. With the brass instruments however, the original plan was to omit the experiment entirely, because it is not possible to carry it out in the same manner as for the woodwind instruments. When the writer discovered an alternate method of carrying out the experiment, several surprises awaited him. These are discussed in each subsection separately and summarized in Section V. Section V, therefore, represents the second purpose of this work: the compiling of results of the experiments, followed by the drawing of conclusions, which lead to a claim and formation of theories.

A few words are in order about the originality of this thesis. With regard to this, the writer can categorically assert that he was unable to find any written material which attempts to answer all three of the questions A), B), and C) posed above. In fact, he was unable to find anything whatsoever which discusses question C) to any greater depth than, for example, that of a comparison of the trumpet to the french horn. What was to be found was discovered almost exclusively in "The Acoustical Foundations of Music" by Dr. John Backus of the University of Southern California, the source which is quoted in this thesis more than any other. The other written sources which are readily available are treatises of one type or another. They present an exhaustive study of one instrument, and <u>do</u> <u>not</u> compare or contrast even two families of instruments, let alone two individual instruments. This thesis may well be considered the opposite of a treatise in that, contrary to discussing any one instrument in depth, it discusses nine instruments generally. One consequence of this is that the corresponding minor instruments (bass clarinet, piccolo, english horn, and contrabassoon) are not discussed at all because the remarks made about the major instruments are applicable to these instruments as well.

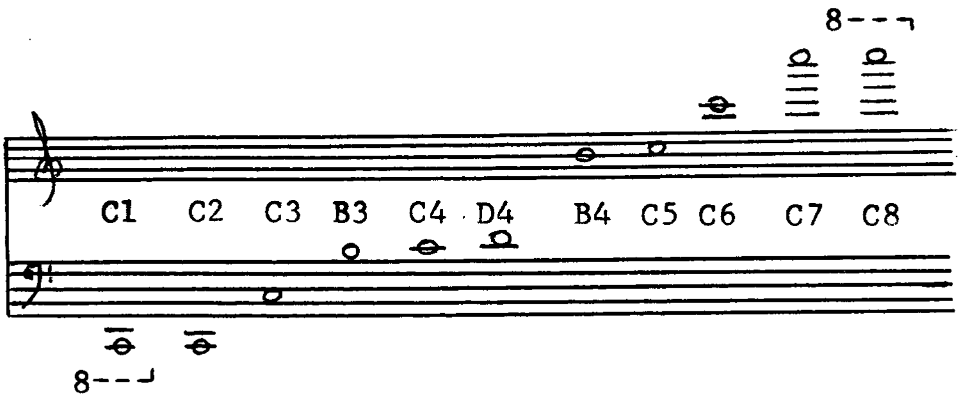
The situation just described led to the following consequences:

1) Relatively few written sources (besides that of Dr. Backus) were of much use in the writing of the thesis. This was especially true in the cases of the Saxophone and the Tuba (see Bibliography.)

2) Interviews with (and demonstrations by) students and professors of the nine instruments discussed were of the greatest importance. This was especially true with regard to the testing of the equation $v = f\lambda$ (see Acknowledgements.)

3) The main source of reference for the formulas which constitute the mathematical and physical basis for the thesis was not a written publication, but the teaching of Dr. Roger Howard of the Department of Physics, University of British Columbia (see Acknowledgements.)

As a point of clarification, it should be explained here that the system of pitch and octave designation used throughout this paper is the USA standard octave notation:



This system was also chosen because it is the one used by Prof. John Backus in his book, "The Acoustical Foundations of Music", which served as an invaluable reference in the writing of this thesis. The frequencies of all pitches from C0 to B8 inclusively are given in the Appendix on page 257.

ACKNOWLEDGEMENTS

I hereby wish to acknowledge the aid of and express thanks and appreciation to the following musicians for assisting me in the experiments conducted for this thesis and for the valuable critiques and suggestions which they offered -- and which I followed.

Clarinettists Prof. Henry Ohlman, Mrs. Debbie Smith;

Flautists Prof. Paul Douglas, Miss Barbara Kallaur;

Oboists Prof. John Chappell, Mr. Bob Pritchard;

Bassoonists Prof. Cortland Hultberg, Mrs. Elodie Kutarna;

Saxophonists Prof. Dave Branter, Mr. Paul Cram;

Trumpeters Prof. Martin Berinbaum, Mr. James Littleford;

French Hornists Prof. Brian G'Froerer, Miss Diana Bos;

Trombonists Prof. R. Sharman King; Mr. Brian Gilbey;

Tubists Prof. Dennis Miller, Mr. Rick Govier.

In addition, I would like to express my deep thanks and gratitude to my thesis adviser, Prof. Cortland Hultberg of the Department of Music, UBC, for his optimistic attitude, scrupulous proofreading, constructive criticism, invaluable suggestions, and overall guidance in the preparation of this thesis.

Finally, I am greatly indebted to Dr. Roger Howard of the Department of Physics, UBC, who first introduced me to the area of acoustics of woodwind and brass instruments, and from whom the ideas for this work originated. I am especially grateful to Dr. Howard for his patient and thorough teaching of the physics which constitutes a solid basis for this paper. Without his voluntary willingness to sacrifice much extra time and trouble, this thesis could not have been written.

Joseph M. Krush,
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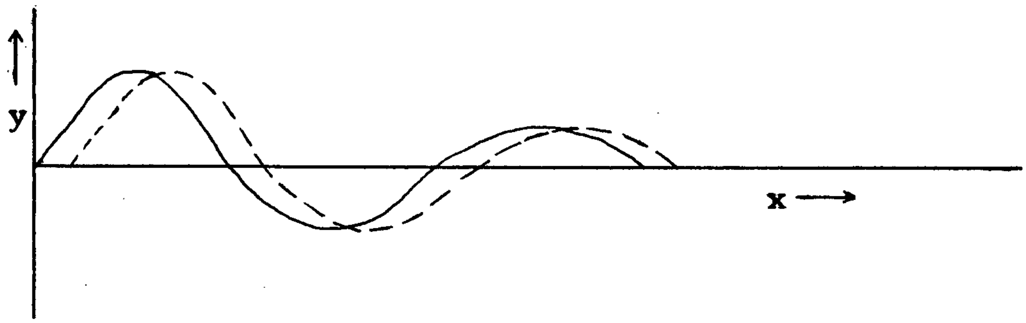
April, 1978.

I. DIFFERENTIAL EQUATIONS OF WAVE-MOTION [1]

The study of acoustics of instruments is closely affiliated with the study of sound itself. Consequently, a practical point of departure in this study is an elementary examination of waves in general and sound waves in particular. A wave is defined as a progressive disturbance propagated from point to point in a medium or in space (without progress or advance of the points themselves).[2] There are many kinds of waves, but all may be classified as being of one or the other of the two basic types: the pulse or the regular train of waves. In the pulse, a single wave which has been created by a single event travels throughout a medium. An example of a pulse is seen when a stone is dropped on the surface of a tank of water. A regular train of waves is one in which there is a regular creation of successive pulses spaced at regular intervals. An example of this is seen when a floating cork is pushed up and down regularly on the surface of a tank of water. The nature of a propagation by the pulse or wavetrain may be a displacement, as in the above examples, or it may be a density fluctuation, as in the case of waves passing through gases such as air. Waves may be divided according to the nature and direction of motion of the elements of the medium transmitting the wave. If a wave is transmitted by oscillations to and fro along the direction of propagation of the wave, it is called a longitudinal wave. The wave transmitted by a stretched string when it is plucked makes a lateral displacement of the elements of the string and is called a transverse wave. If a metal bar is gripped firmly at one end and then twisted at the other, upon release of the tension the rod will execute small circular arcs about its axis, transmitting torsional waves.

Sound waves fall into the general category of wave trains (a term synonymous with "regular train of waves"), and are longitudinal waves whose nature of propagation is through density fluctuation of air molecules. This study will deal almost exclusively with waves of this nature. The behavior of sound waves will be examined under the following different conditions: 1) waves traveling through space, 2) waves traveling through a medium (in particular an air column), which may be either a) a cylindrical tube (open or closed), or b) a conical tube (full or truncated.)

A regular train of waves traveling to the right along the x-axis with a velocity v may be represented graphically as follows:



The solid line represents the original position of the wave, and the dotted line represents its position a short time later. The shape of the wave itself undergoes no change whatsoever.

The above is represented analytically by the expression

$$y = f(vt - x), \qquad (1)$$

where f is some function of x (position) and t (time), and y is the magnitude of the condition which is changing. Suppose at some point $(x + x_1)$ and some subsequent time $(t + \tau)$, y has the same value as in (1) above, then

$$y = f[v(t + \tau) - (x + x_1)] \,. \qquad (2)$$

Thus y will remain unchanged provided

$$v = \frac{x_1}{\tau} \qquad (3)$$

because

$$(1) \quad y = f\left(\frac{x_1 t}{\tau} - x\right) = f\left(\frac{x_1 t}{\tau} + x_1 - x - x_1\right) = y. \qquad (2)$$

We may regard the singularity of magnitude y at (x, t) as having been conveyed to $(x + x_1)$ at time $(t + \tau)$, that is, of having traversed the distance x_1 in the time τ. Thus v in equation (3) represents the velocity of the wave. Similarly, $y = f(vt + x)$ represents a wave traveling to the left along the x-axis with velocity v.

The most important type of sound wave is the simple harmonic wave in which the periodic change follows the symmetrical sine or cosine law. Here, y has a maximum positive value "a" 1) at regular intervals of t if we fix x, 2) at regular intervals of x if we fix t. Also, as we proceed along the sine or cosine wave, for each positive maximum of y we encounter two zero values, one corresponding to y increasing and the other to y decreasing. The equation for a sine wave is of the form

$$y = A\sin 2\pi\left(\frac{t}{\tau} - \frac{x}{\lambda}\right) \qquad (4)$$

where: 1) if we put x = 0, y passes through zero for y increasing at regular intervals of time τ, and 2) if we put t = 0, y passes through zero for y increasing at regular intervals of distance λ. The quantities τ and λ are termed respectively the <u>period</u> and <u>wave-length</u> of the wave motion. The wave-length is defined as the distance between successive crests or successive troughs, or between alternate extrema (maxima and minima) of the wave-motion. Further, if we write equation (4) in the form

$$y = A\sin\frac{2\pi}{\lambda}(\lambda\frac{t}{\tau} - x) \qquad (5)$$

and compare the coefficient of t with that in equation (1), we see that the velocity v of propagation of the wave is given by

$$v = \frac{\lambda}{\tau}\ . \qquad (6)$$

The reciprocal of the period τ is termed the <u>frequency</u> f of the wave-motion, and hence equation (6) may be written

$$v = f\lambda. \qquad (7)$$

This equation, connecting wave-length, frequency, and velocity of propagation, is of fundamental importance in wave-motion, and will be used throughout this paper. If we rewrite equation (5) in the form

$$y = A\sin\frac{2\pi}{\lambda}(vt - x) \qquad (8)$$

and differentiate (8) twice with respect to x, we obtain:[3]

$$\frac{\partial y}{\partial x} = \frac{2\pi}{\lambda} A\cos \frac{2\pi}{\lambda} (vt - x)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{-2\pi}{\lambda} A\sin\frac{2\pi}{\lambda}(vt - x)(\frac{2\pi}{\lambda}).$$

But $A\sin\frac{2\pi}{\lambda}(vt - x) = y$, $\therefore \frac{\partial^2 y}{\partial x^2} = \frac{-4\pi^2}{\lambda^2} y.$ (9)

Now we differentiate (8) twice with respect to t to obtain:

$$\frac{\partial y}{\partial t} = \frac{2\pi}{\lambda} A\cos\frac{2\pi}{\lambda}(vt - x)v$$

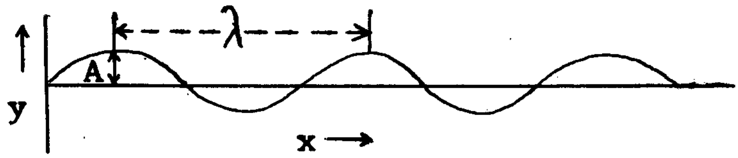
$$\frac{\partial^2 y}{\partial t^2} = \frac{-2\pi}{\lambda} A\sin\frac{2\pi}{\lambda}(vt - x)(\frac{2\pi}{\lambda})v^2 .$$

But $A\sin\frac{2\pi}{\lambda}(vt - x) = y$, $\therefore \frac{\partial^2 y}{\partial t^2} = \frac{-4\pi^2}{\lambda^2} v^2 y.$ (10)

Now from equations (9) and (10) we obtain:

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} . \qquad (11)$$

Equations (9), (10), and (11) -- especially (11) -- are called differential equations of wave-motion, and when we come across an equation of the type of equation (11), we know that a wave-motion must be involved. Equation (11) has the important feature that the coefficient of the derivative on the right-hand side represents the square of the velocity of the wave; this means that we do not need to solve the equation to obtain the velocity of propagation. The coefficient "A" in equations (4), (5), and (8) is called the <u>amplitude</u> of the wave. The amplitude A is defined to be the absolute value of the maximum displacement from a zero value during one period of an oscillation. The following is a diagram of a simple harmonic wave:



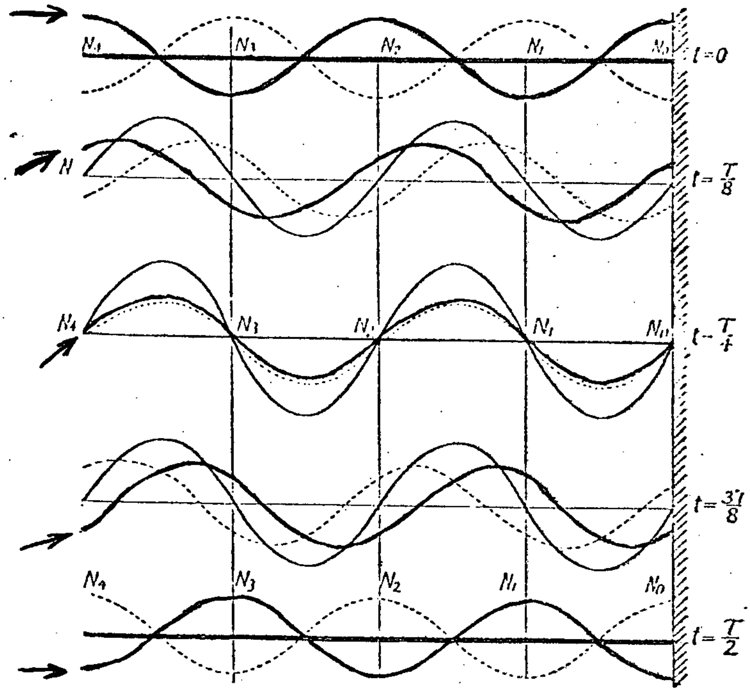
If instead of equation (8) we take

$$y = A\sin\frac{2\pi}{\lambda}(vt - x) + B\sin\frac{2\pi}{\lambda}(vt + x), \qquad (12)$$

we again obtain equation (11), which may represent a combination of two waves traveling in opposite directions along Ox with velocity v.

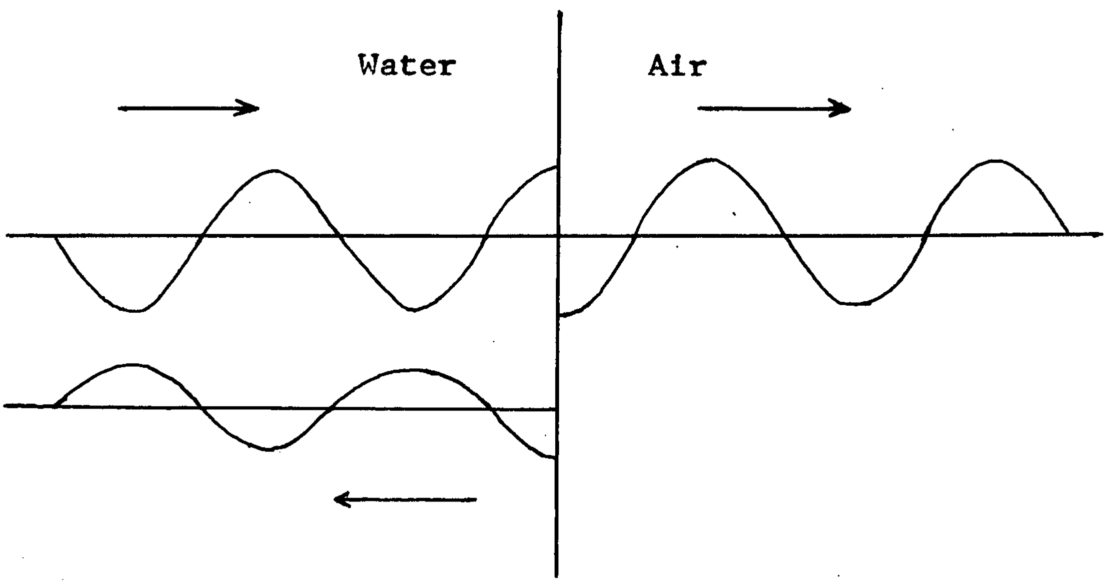
If more than one train of waves is present, the total displacement of the medium is obtained by superimposing the displacements which would be caused by each wave train acting alone. The final result will depend upon whether the vibrations of the wave trains are in the same or in different planes. If the planes are the same, the waves interfere with each other. If two wave trains of the same wave-length and amplitude, and vibrating in the same plane, are in phase, that is, if two wave-crests, two wave-troughs, or two corresponding points on the wave surface, arrive at a given place simultaneously, the displacement of the medium is twice that which would occur with one wave train acting alone. Conversely, if the wave trains are completely out of phase, that is, if a wave-crest from one wave train arrives simultaneously with a wave-trough from another, the two displacements of the medium are equal and opposite, and the total effect is zero displacement; this is due to destructive interference.

The waves discussed thus far have been progressive waves, i. e. waves spreading outwards continuously from some source so that small portions of the medium oscillate up and down (transverse waves) or to and fro (longitudinal waves) as the waves pass by. On the other hand, if the medium is limited, for example by a rigid barrier, reflected waves are produced which interfere with the oncoming waves. The effect of the interference is remarkable in that the progressive nature of the waves vanishes. At certain regular distances from the reflecting barrier the medium is perpetually at rest; these points are called nodes. There are also other regularly spaced points, called antinodes, at which the amplitudes have maximum, though fluctuating, values. Since the portion of the medium immediately in contact with the rigid barrier must be at rest, the amplitudes produced by the incident and reflected waves at that point must be equal and opposite. Reflection of waves at a denser or more rigid medium therefore results in a change of phase of π with respect to the incident wave. The formation of stationary or standing waves is shown in the figure on the following page. The incident wave, represented by the blue line, travels from left to right. It meets the rigid barrier on the right side and gives rise to a reflected wave represented by the dotted line traveling from right to left. The total displacement of the medium or the standing



wave which results from the interference of the incident and reflected waves is shown by the black line. The points N_1, N_2, N_3 (the nodes) are observed to be perpetually at rest, whereas points midway between these points (the antinodes) undergo the maximum displacement. It is important to note that the wave-length is twice the distance between consecutive nodes.

On the other hand, if a progressive wave passes from a denser to a less dense medium, the displacement of the less dense medium under an incident compression is greater than the displacement of the denser medium. At the boundary, therefore, the denser medium moves into the less dense medium for a short distance, and thus a rarefaction is created in the denser medium close to the boundary. This results in a reflected wave of rarefaction. Since the boundary is free to move, the boundary becomes an antinode and the waves are reflected <u>without change of phase</u>. An example of this sort of wave propagation is from water to air, as is illustrated below:



The analytical examination of stationary waves produced by reflection without change of phase is as follows. The total displacement y of the medium at any point x from some arbitrary origin 0 is the sum of the two displacements y_1 and y_2 produced by the incident and reflected waves respectively. Hence

$$y = y_1 + y_2 = A\sin\frac{2\pi}{\lambda}(vt - x) + A\sin\frac{2\pi}{\lambda}(vt + x)$$

$$= A[(\sin\frac{2\pi}{\lambda}vt)(\cos\frac{2\pi}{\lambda}x) - (\cos\frac{2\pi}{\lambda}vt)(\sin\frac{2\pi}{\lambda}x)] +$$

$$A[(\sin\frac{2\pi}{\lambda}vt)(\cos\frac{2\pi}{\lambda}x) + (\cos\frac{2\pi}{\lambda}vt)(\sin\frac{2\pi}{\lambda}x)]$$

$$= 2A \cdot \sin\frac{2\pi}{\lambda}vt \cdot \cos\frac{2\pi}{\lambda}x \ . \qquad (13)$$

From equation (13) we want to find when the displacement y is zero for all values of t. We find, therefore, all values of x for which $\cos\frac{2\pi}{\lambda}x = 0$. Therefore $\frac{2\pi}{\lambda}x$ must equal $(2n - 1)(\frac{\pi}{2})$ for any integer n. Therefore $x = (2n - 1)\frac{\lambda}{4}$ for any integer n.

Conclusion: For all values of t, at distances given by $x = (2n - 1)\frac{\lambda}{4}$, where n is an integer, the displacement y is zero. These points clearly represent the nodes of the stationary wave system and the distance between successive nodes is half the wavelength.

In the case of waves propagated by a density fluctuation of air, the source of sound consists of some mechanically vibrating system such as a vibrating strip. As the strip moves up, air which is in contact with the strip on its upper surface is compressed. This compression causes a local rise in pressure which compresses in turn the air immediately above it. The process continues in this way, with the result that a compression pulse spreads through the air. As the strip moves downwards, a slight vacuum or rarefaction is created in the wake of the moving strip. Consequently, air moves from neighbouring points to occupy this vacuum, and therefore a rarefaction pulse spreads through the medium following the preceding compression pulse. Succeeding vibrations of the strip create a series of compressions and rarefactions following each other in regular succession. Such a series constitutes a sound wave. It is detected by the ear, which possesses a delicate membrane which is set vibrating by the successive compressions and rarefactions which constitute the sound wave. If the frequencies of the vibrations are above or below the human hearing threshold, they are undetectable by the ear.

II. THE CYLINDRICAL TUBE

Boundary Conditions for the Cylindrical Tube [1]

There is a total of four possible boundary conditions for the cylindrical tube:

1) tube open at end one, closed at end two,
2) tube open at end two, closed at end one,
3) tube closed at both ends,
4) tube open at both ends.

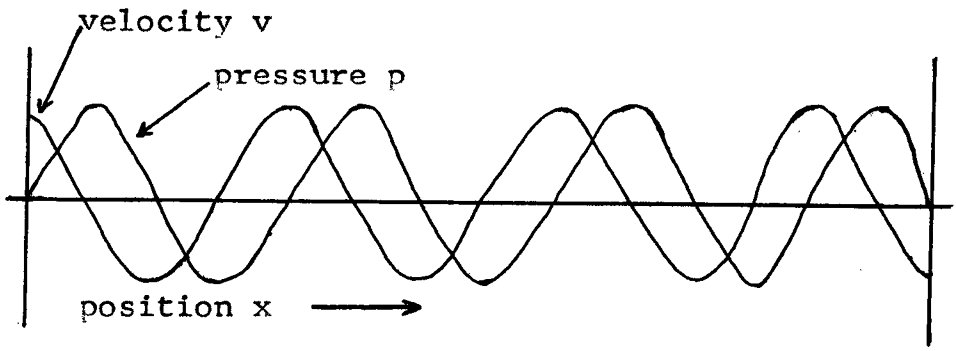
From the musician's standpoint, case 1 equals case 2 and case 3 has no application, because no sound is emitted from a tube closed at both ends and without any side holes. Only cases 1 and 4, therefore, will be considered here. The two tubes will be taken to be of equal length (i. e. $L = L'$); the clarinet will be used as an example of case 1, and the flute as an example of case 4.

We start, now, with the general equations for pressure p and velocity v:

$$p = A\sin(kx) + B\cos(kx) \qquad (1)$$

$$v \propto \frac{\partial p}{\partial x} = kA\cos(kx) - kB\sin(kx) \qquad (2)$$

Case 1: We know that if we close one end of a tube, the molecular velocity v at that end equals zero. From the following graph, we may conclude, that with v = 0, the pressure p at that end is at a maximum, just as when $p \doteq 0$, v is at a maximum:



To impose boundary conditions, we first put x = 0 in equation (2) above, thus giving

$$v \propto \frac{\partial p}{\partial x} = kA\cos(kx).$$

Therefore $v \propto \frac{\partial p}{\partial x} = kA$, therefore A = 0. Equation (1) now becomes:

$$p = B\cos(kx) \qquad (3)$$

Next, we put x = L, where L represents the other (open) end of the pipe. Equation (3) now becomes:

$$p = B\cos(kL).$$

We know that at the open end of a tube, $p \doteq 0$,

$$\therefore B\cos(kL) = 0$$

$$\therefore kL = (2n - 1)\frac{\pi}{2} \text{ for any integer } n$$

$$\therefore k = \frac{(2n - 1)\frac{\pi}{2}}{L} \text{ for any integer n.} \qquad (4)$$

This is the value of k which is fixed by the boundary conditions. On the other hand, we expect a simple harmonic wave to be in the form of:

$$p = B\cos\frac{2\pi}{\lambda}x. \qquad (5)$$

Therefore by definition in terms of wavelength,

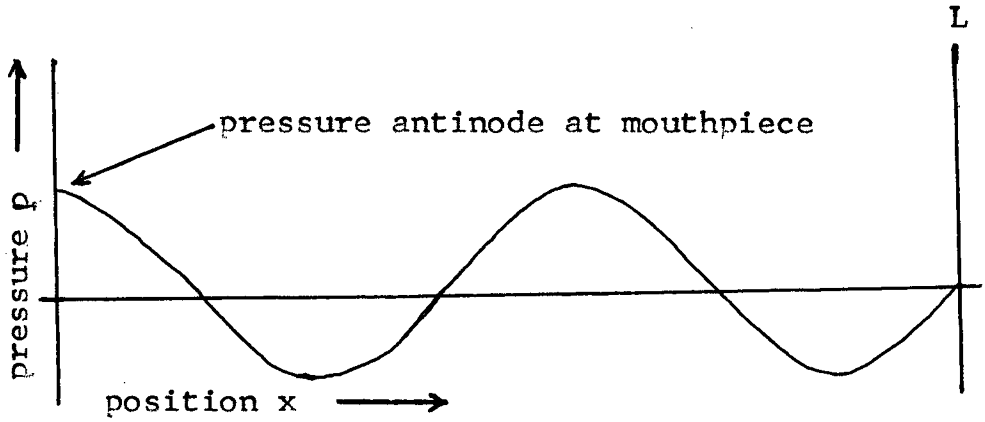
$$k = \frac{2\pi}{\lambda} \qquad (6)$$

Now, combining equations (4) and (6) we get:

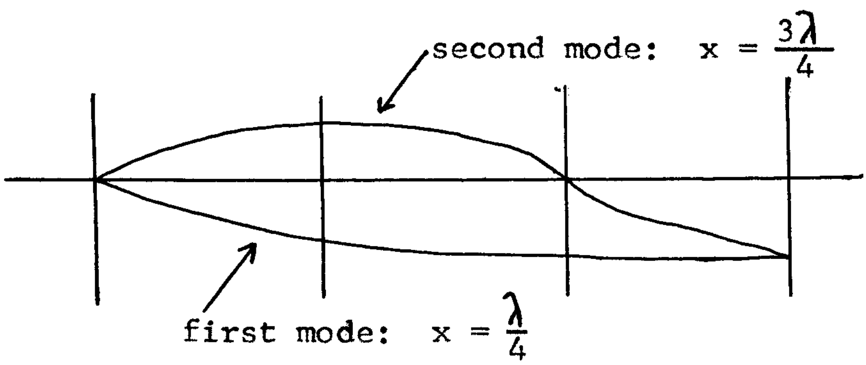
$$\frac{(2n - 1)\frac{\pi}{2}}{L} = \frac{2\pi}{\lambda} .$$

Therefore $L = (2n - 1)\frac{\lambda}{4}$ for any integer n, or $L = \frac{n\lambda}{4}$ for odd n. Now we may find all values of x for which p = 0 by finding all values of x for which $\frac{2\pi}{\lambda}x = (2n - 1)\frac{\pi}{2}$ for any integer n. Therefore $x = \frac{n\lambda}{4}$ for odd integers n, or $x = (2n - 1)\frac{\lambda}{4}$ for any integer n. This means that p = 0 when $x = \frac{\lambda}{4}$, $\frac{3\lambda}{4}$, $\frac{5\lambda}{4}$, $\frac{7\lambda}{4}$, etc... Consequently, the general equation involving λ of the cylindrical tube of length L closed at one end (here the clarinet) is, as stated above, $L = \frac{n\lambda}{4}$ for odd n. In the diagram on the next page, n is assigned the value of 7.

Cylindrical tube closed at one end:



Since $L = \frac{n\lambda}{4}$ must be satisfied, $\lambda_{max} = 4L$. As stated above, $p = B\cos\frac{2\pi}{\lambda}x$. Consequently, $p = 0$ beginning at $\frac{\pi}{2}$ every 180°.



Case 4: (Using x', p', λ', k', and L' for x, p, λ, k, and L.) We know that at an open end of a tube, $p' \doteq 0$. Consequently, to impose boundary conditions, we first put $x' = 0$ in equation (1) above and get:

$$p' = B\cos(k'x'), \therefore B = 0.$$

Now equation (1) becomes:

$$p' = A\sin(k'x') \, . \tag{7}$$

We next put $x' = L'$, where L' represents the other open end of the pipe. Equation (7) now becomes:

$$p' = A\sin(k'L')$$

$$\therefore \; A\sin(k'L') = 0$$

$$\therefore \; k'L' = m\pi \quad \text{for any integer } m,$$

$$\therefore \; k' = \frac{m\pi}{L'} \quad \text{for any integer } m. \tag{8}$$

This is the value of k' which is fixed by the boundary conditions. On the other hand, we expect a simple harmonic wave to be in the form:

$$p' = B\cos\frac{2\pi}{\lambda'} \, . \tag{9}$$

Therefore by definition in terms of wavelength,

$$k' = \frac{2\pi}{\lambda'} \, . \tag{10}$$

Now, combining equations (8) and (10) we get:

$$\frac{m\pi}{L'} = \frac{2\pi}{\lambda'} \, , \; \therefore$$

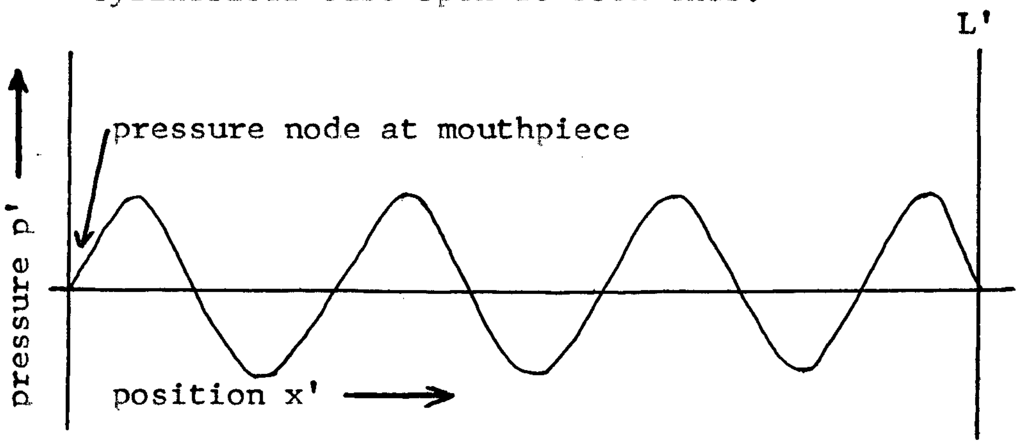
$$L' = \frac{m\lambda'}{2} \quad \text{for any integer } m.$$

Now we may find all values of x' for which $p' = 0$ by finding all values of x' for which $\frac{2\pi}{\lambda'}x' = m\pi$ for any integer m. Therefore $x' = \frac{m\lambda'}{2}$ for any integer m. This means that $p' = 0$ when $x' = 0$,

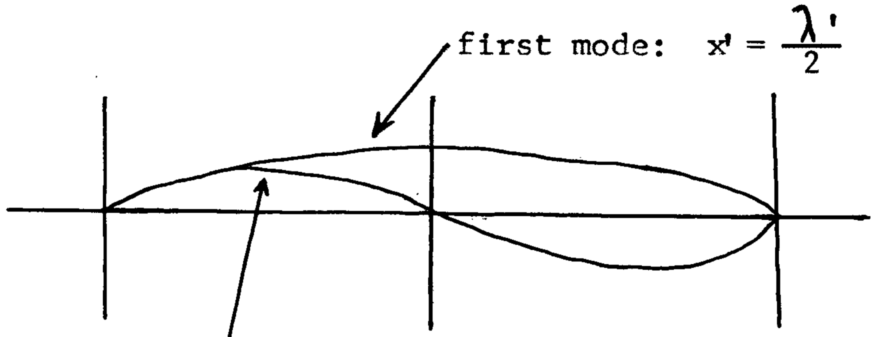
$\frac{\lambda'}{2}$, λ', $\frac{3\lambda'}{2}$, $2\lambda'$, $\frac{5\lambda'}{2}$, $3\lambda'$, $\frac{7\lambda'}{2}$, $4\lambda'$, etc...

The general equation involving λ' of the cylindrical tube of length L' open at both ends (here the flute) is $L' = \frac{m\lambda'}{2}$ for some integer m. In the diagram below, m is assigned the value of 7.

Cylindrical tube open at both ends:



Since $L' = \frac{m\lambda'}{2}$ must be satisfied, $\lambda'_{max} = 2L'$. As stated above, here $p' = A\sin\frac{2\pi x'}{\lambda'}$. Consequently, $p' = 0$ beginning at 0 every 180°.



Comparing the flute to the clarinet, we see that the fundamental frequency of the clarinet is half that of the flute, although the two instruments are of nearly equal length:

	Clarinet	Flute
Length:	670 mm	618.5 mm
Lowest possible pitch:	E3 (sounding D3, a major second lower)	C4 (a minor seventh above D3)

Possible Frequencies

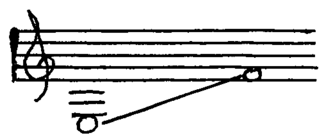
The Clarinet

Since the cylindrical tube closed at one end vibrates with modes that are odd multiples of the fundamental, the second mode of oscillation of the clarinet has a frequency three times that of the fundamental:

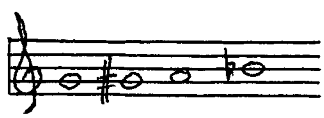
$$p = 0 \text{ first at } x = \frac{\lambda}{4}, \text{ next at } x = \frac{3\lambda}{4}.$$ [2]

Musically, this results in a rise in pitch of an octave plus a perfect fifth, i. e. a perfect twelfth. This is why a clarinet overblows the twelfth, and not the octave; consequently, it becomes necessary to fill in the gap between the seventh and the twelfth with additional holes and keys. By "additional holes" is meant holes other than the original six tone holes which suffice to produce the first seven notes of the scale.

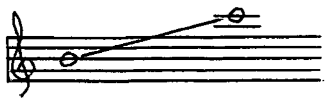
The gap in the scale is filled in the following way: the lowest note of the Bb clarinet is E3, sounding D3, a whole tone lower. The lowest note of the basic scale is G3, a minor third above E3. By providing two additional holes and keys, the two intervening semitones, F3 and F#3, are supplied. The basic scale is now ascended from G3 by opening the six holes in turn (with additional keys provided for sharps and flats), until F4 is reached. This pitch range is known as the low (chalumeau) register:



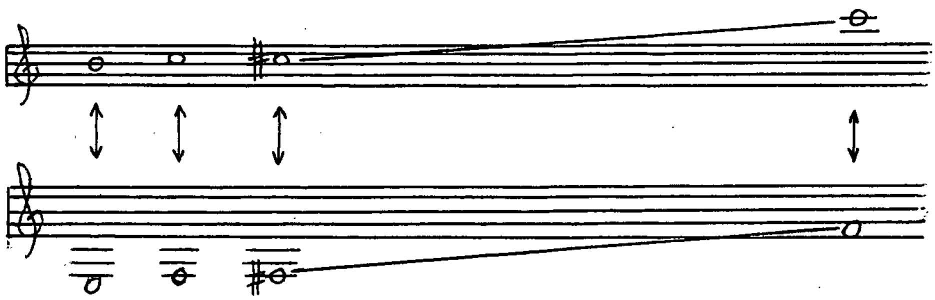
To "fill in" the notes from F#4 to Bb4, additional holes, provided closer to the mouthpiece, must be opened. These additional holes (and keys) make clarinet fingerings somewhat more complicated than those of other woodwinds. This "linking" type of register on the clarinet is known as the throat register:



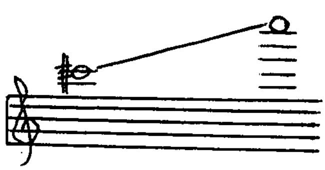
Between Bb4 and B4 is found what is known as the first "break", i. e. the point at which the first (and most important) change in register occurs. To produce B4, all the holes must again be closed (so that the fingering is the same as for E3 plus the thumb key), and the instrument is overblown to produce the next resonance mode a twelfth above the original. (This is accomplished by opening the vent-hole, located on the side of the clarinet about 15 cm down from the mouthpiece tip.) With the vent-hole open, the clarinet operates in the second mode, producing the clarion register:[3]



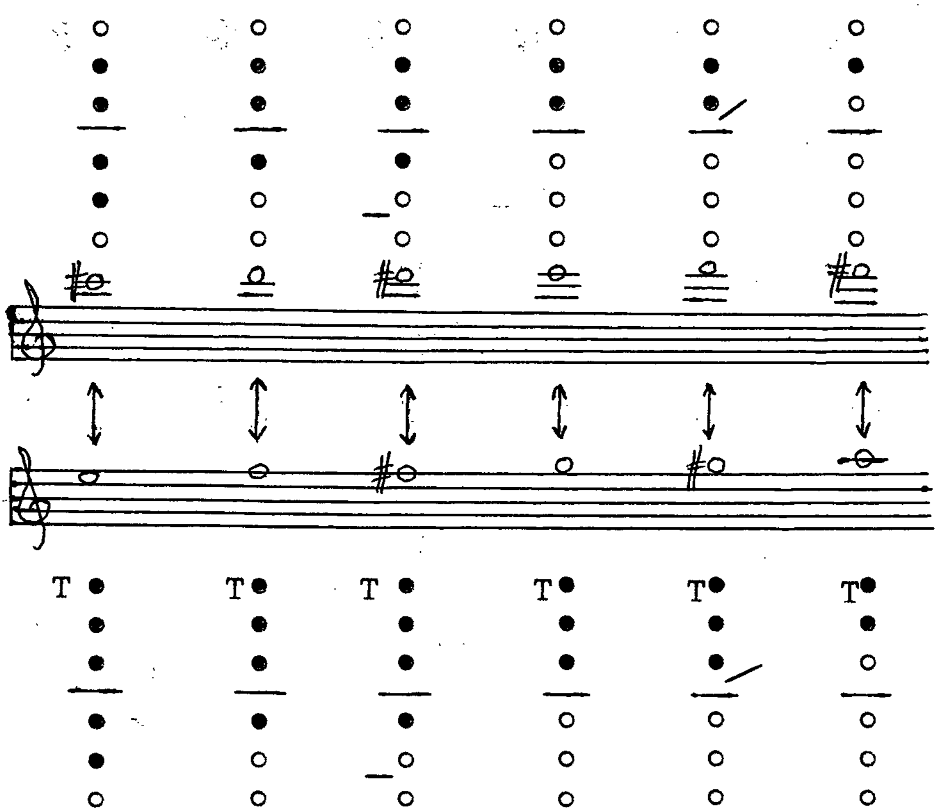
The corresponding fingerings between the two registers are as follows:



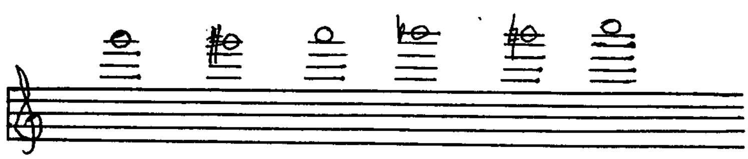
The next "break" occurs between C6 and C#6; from C#6 up is found the extreme register of the clarinet:



We recall now, that p = 0 at $x = \frac{\lambda}{4}$, $x = \frac{3\lambda}{4}$, and next at $x = \frac{5\lambda}{4}$. Now just as $\frac{3\lambda}{4}$ resulted in a frequency three times that of the fundamental, similarly $\frac{5\lambda}{4}$ results in a frequency five times that of the fundamental, i. e. two octaves plus a major third above it. But this may also be regarded as $\frac{5}{3}(\frac{3\lambda}{4})$, where $\frac{3\lambda}{4}$ is the previous mode. It is in this way -- by overblowing the corresponding pitches a major sixth below, that most of the pitches of the extreme register are produced. Thus, the following correspondences result: [4]

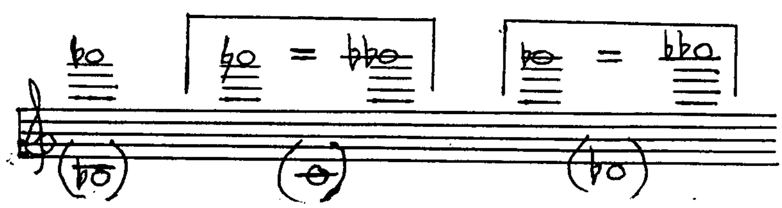


It is important to realize that these pitches, when produced by overblowing, are not always well in tune in equal temperament, because the ratio 5/3, when used to produce a pitch a major sixth higher by overblowing, produces the pitch in the just scale -- not in the equal-tempered scale. For this reason, clarinettists do not usually use the fingering system shown above; instead, pitches which are in tune in the equal-tempered scale are produced by use of additional keys and holes. The fingerings above are so designated for purposes of examination of the clarinet from the acoustical standpoint. For the six highest pitches which may be produced on the clarinet, G6 to C7,

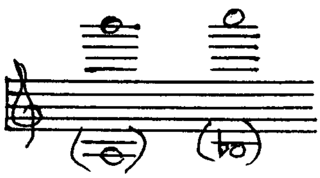


the fingering (and thus the manner of producing the pitch) is not as consistent as it is in the chalumeau and clarion registers. Some modern writers give as many as 40 fingerings for these pitches; G6 alone has 10 or 12. For purposes of study from the acoustical standpoint, we examine here the fingerings which come about through the method of derivation used hitherto. The next note, G6, can still be fingered in the same way as the previous six pitches, by overblowing the major sixth below.

Just as this frequency is the result of $\frac{5\lambda}{4}$, i. e. five times the fundamental frequency, similarly, $\frac{7\lambda}{4}$ results in a frequency seven times that of the fundamental, i. e. theoretically two octaves and a minor seventh (= a twenty-first) above it. We know, however, that in the harmonic series the seventh harmonic is always flatter than the corresponding pitch in equal temperament. In this high a register, this is especially so, to the extent that the twenty-first is so flat that it equals the equal-tempered twentieth. Thus, the Ab6 (= the flat A♮6) and the A6 (= the flat Bb6) and the Bb6 (= the flat B♮6) may be produced by overblowing the twenty-first (= 2 octaves plus an extremely flat minor seventh):[5]



Finally, just as $\frac{7\lambda}{4}$ results in a frequency seven times the fundamental, similarly $\frac{9\lambda}{4}$ results in a frequency 9 times the fundamental, i. e. three octaves and a major second (= a twenty-third) above it. It is by overblowing the twenty-third (i. e. the twelfth twice over), that the two highest pitches of the clarinet, B6 and C7, may be produced:[6]



It must be stressed here again, that the pitches produced by overblowing in this high a range are not always in tune in the equal-tempered scale. Often, clarinettists use alternate fingerings which result in better tuned pitches. The analogies which are made here are made in order to make an examination of harmonics and overblowing from the acoustical standpoint.

Testing the Equation $v = f\lambda$[7]

It is interesting to test the equation $v = f\lambda$ in a practical situation involving the clarinet. We know that the velocity v of sound is approximately 33145 cm/sec at 25° C., and that it increases by 59 cm/sec for each degree C above 25° C.[8] Therefore at body temperature, 37° C, the velocity v of sound is approximately 33145 + 59(12) = 33853 cm/sec. We know also that in the case of the clarinet of length L, $L = \frac{n\lambda}{4}$ must be satisfied, therefore $L_{max} = 4L$ (see page 17). The total length of the clarinet is 66.8 cm, therefore $\lambda_{max} = \lambda$ for the lowest pitch of the clarinet = 66.8(4) = 267.2 cm. Thus, from the equation $v = f\lambda$, we get: 33853 cm/sec = f(267.2 cm), ∴ f = 126.7 cycles per second = 126.7 hertz (hz).

The closest frequency to this in the equal-tempered scale is that of B2, whose frequency is 123.47 hz. In reality, the lowest pitch of the clarinet is the written E3, which sounds as D3, a minor third above B2. The explanation for this discrepancy is that not only is the end correction involved, but so is the entire length of the clarinet. Now the bore of the clarinet is not cylindrical throughout its whole length; the cylindrical section begins about 4 cm from the tip of the mouthpiece and extends about halfway down the bottom joint.

From that point on, it flares outward, assuming a conical shape in the bell. Thus it is understandable that pitches whose production involves the lower half of the bottom joint and the bell will deviate from those which are derived by calculations such as the above, because those calculations presuppose a perfectly cylindrical tube. For further comparison, three more similar calculations were made. In each case, the length L was measured from the tip of the mouthpiece to the centre of the first open hole. The results are summarized in the table on the following page.

We observe from the table, that the higher the pitch, the smaller the discrepancy. This is because the clarinet is, in fact, cylindrical in the top joint, which means that the practical case comes closer and closer to the ideal case as higher and higher pitches are examined. Finally, with all holes open at Bb4 (sounding Ab4), the two are very nearly identical.

1a	Examined Pitch: Written:	E3	G3	F4	Bb4
	Sounding:	D3	F3	Eb4	Ab4
1b	Known frequency of this pitch (hz):	146.83	174.61	311.13	415.3
	Length of pipe (L) (cm):	66.8	41.5	23.8	20.4
	λ (where $\lambda_{max} = 4L$) (cm):	267.2	166.0	95.2	81.6
	Calculated frequency (33853 cm/sec ÷ 4L) (hz):	126.7	203.9	355.6	414.9
2a	Pitch whose frequency in equal temperament is nearest to that obtained above:	B2	Ab3	F4	Ab4
	Known frequency of this pitch (hz):	123.47	207.65	349.23	415.3
	Discrepancy of 2a with respect to 1a (by interval):	- 3rd lower	- 3rd hghr.	+ 2nd hghr.	none
	Ratio of calculated frequency with respect to 1b:	.863	1.17	1.14	.999

The Vent-Hole and the Resonance Curves of the Clarinet [9]

As was mentioned earlier (see page 23), the vent-hole is located about 15 cm below the mouthpiece, and is operated by the thumb key. This is the key which enables overblowing by the twelfth by essentially destroying the lowest resonance. How this happens may be explained by examining the resonance curve for the lowest note of the clarinet, E3:

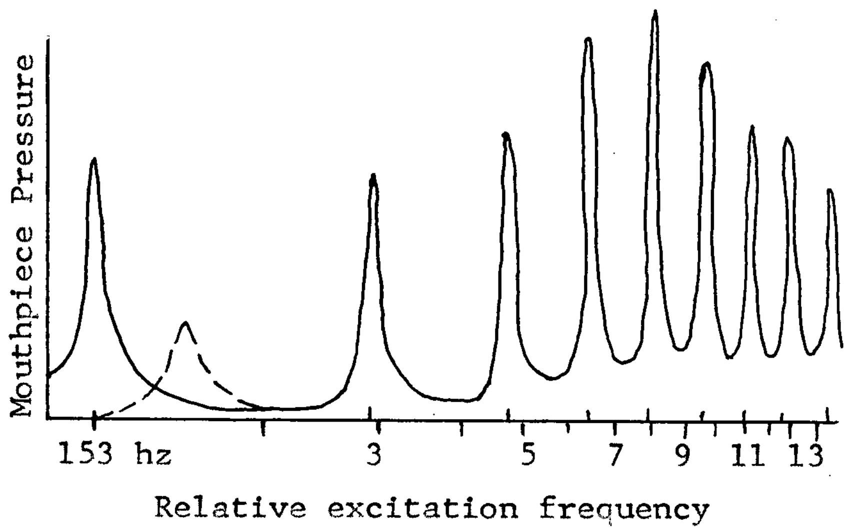
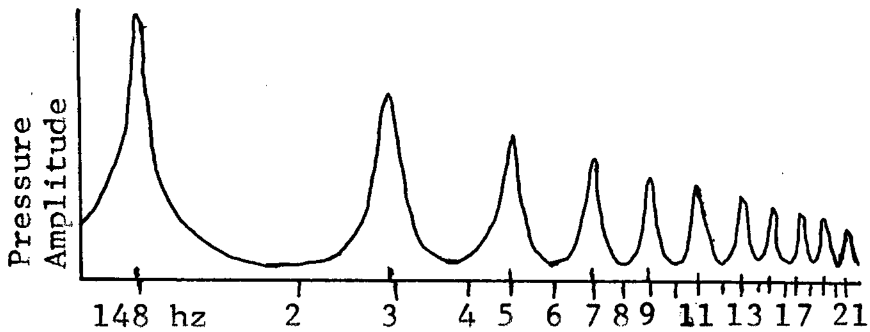


Figure 1

When the vent-hole is opened, the lowest resonance moves from its position as shown by the solid line to that shown by the dotted line. The remaining higher resonances are essentially unchanged.

Let us now examine the resonance curve for a plain metal tube closed at one end:



Relative excitation frequency

Figure 2

The resonance frequencies are quite regularly odd multiples of the fundamental. If a clarinet mouthpiece with reed is placed on this metal tube and the resonances are again measured, the resulting curve is essentially identical to that of Figure 2 above, except for slight (relatively insignificant) changes in the positions of the high resonances (above the eleventh harmonic). These changes are due to the internal shape of the clarinet mouthpiece.

If the reed is blown in the usual way to produce a tone in the metal tube, an internal standing wave is obtained which contains mostly odd harmonics, since their frequencies coincide with the resonance frequencies. The even harmonics have small amplitudes, since their frequencies lie in between the resonance frequencies. The following figure shows the harmonic structure of this standing wave.

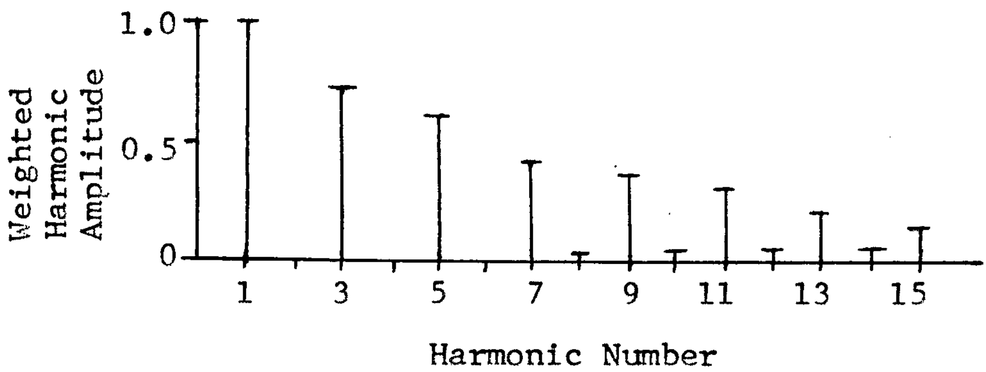


Figure 3

The amplitude of each harmonic has been multiplied by its number to give the weighted harmonic amplitude shown above. For example, the weighted third harmonic amplitude is three times the measured harmonic amplitude, etc... Since it can be shown that the higher the frequency of the harmonic, the better it radiates from the instrument, this weighting gives a more realistic representation of how the ear will appreciate the actual harmonic structure. It also makes the harmonics easier to see on the graph.

If the same resonance measurements are made on the clarinet, it is found that they are considerably distorted as compared with those in the cylindrical metal tube, as we see in Figure 1 above for the note E3 on the clarinet. The resonance frequencies depart more and more from the harmonic frequencies as we go to higher values; for the particular note E3, the eighth harmonic, for instance, coincides more nearly with a resonance than either the seventh or ninth. If the clarinet is blown and the harmonic structure of the internal standing wave is analyzed, we find that the eighth harmonic is more prominent in this tone than the seventh or ninth, as is shown in the following figure:

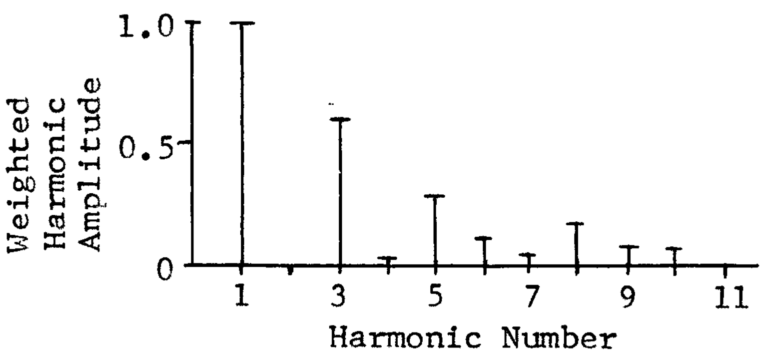


Figure 4

As the scale is ascended on the clarinet, the resonances become fewer in number; for the note Bb4 just before the break to the clarion register, there are only two good resonances, as shown in Figure 5 below:

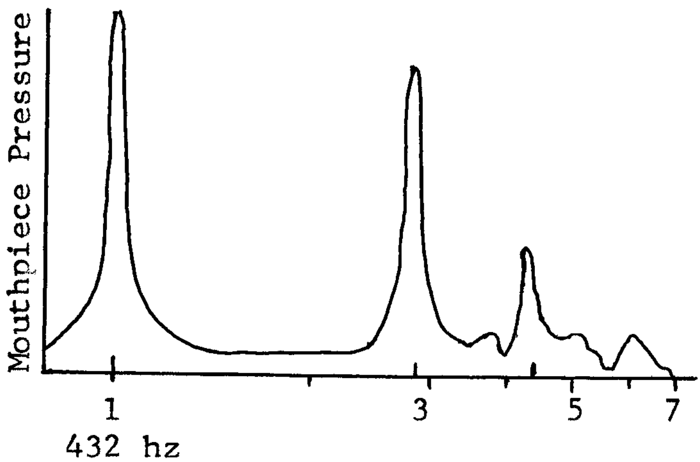


Figure 5

Consequently, the tone has very few strong harmonics, as is shown in the figure below, which shows the weighted harmonic structure of the internal standing wave in a clarinet sounding Bb4.

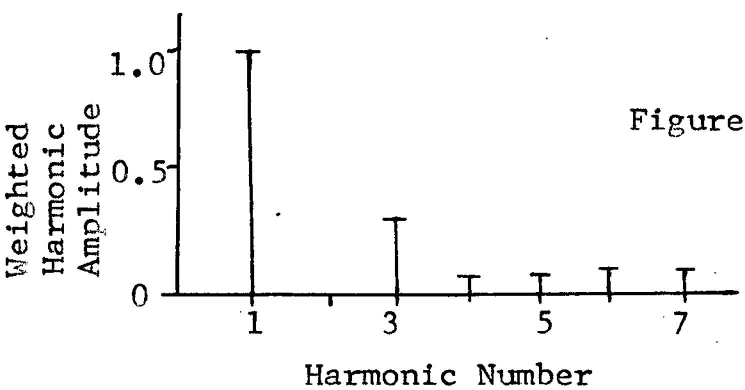


Figure 6

The lack of resonances in this region accounts for the poor quality of the throat register of the clarinet.

In the extreme register, above the break, there is little correspondence between the harmonic frequencies and the resonances, as may be seen in Figure 7 below:

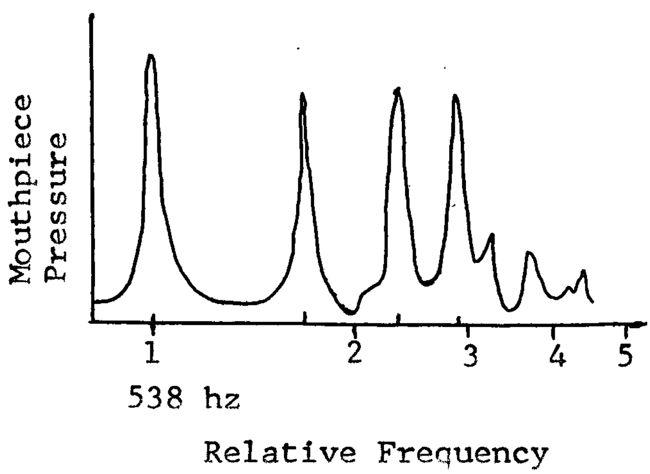


Figure 7

The third harmonic is the only one near a resonance, and it is not very close. Consequently, the tone produced is mostly fundamental, but has both even and odd harmonics, as shown in the following figure:

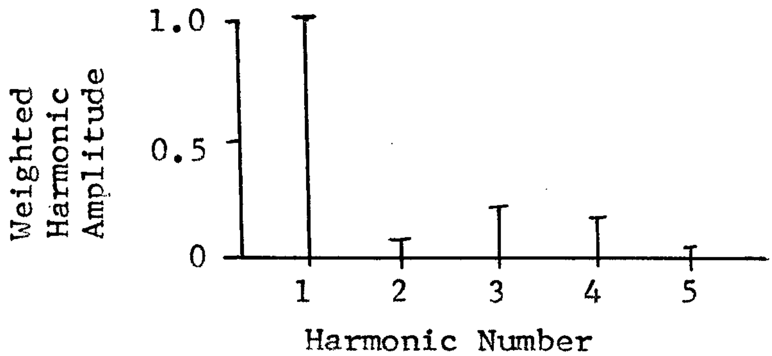


Figure 8

Figure 7 shows the resonance curve for the note D5 on the clarinet; figure 8 shows the weighted harmonic structure of the internal standing wave in a clarinet sounding this note. We see from these two figures why the quality of the clarinet tone is different in the higher register.

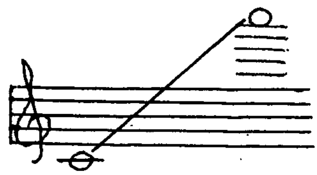
The Flute

Since the cylindrical tube open at both ends vibrates with modes that are integral multiples of the fundamental, the second mode of oscillation of the flute has a frequency twice that of the fundamental:

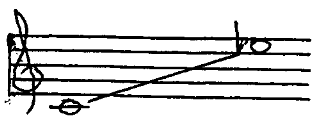
$p' = 0$ first at $x' = 0$, next at $x' = \frac{\lambda'}{2}$, next at $x' = \lambda'$.[10]

Musically, this results in a rise in pitch of an octave, which is the basic interval by which the flute overblows. This property, plus the fact that the flute has a pressure node (and not an antinode) at the mouthpiece, makes the construction and fingerings of a flute much less complicated than are those of the clarinet. There are no "gaps" in the scale which need to be filled in due to overblowing by a twelfth; consequently, the flute has considerably less "additional" holes and keys.

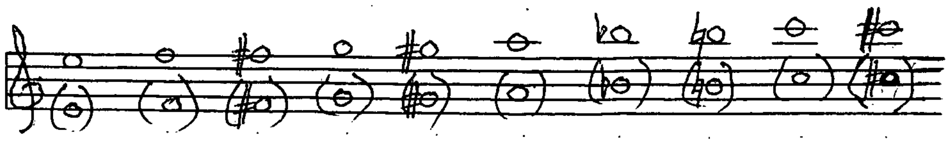
The basic scale of the flute is the six-hole, seven-note scale described earlier; it starts at D4 with the six holes closed. Additional holes and keys are used to provide the sharps and flats and to extend the range down to C4.[11] C4 is produced with all holes closed; with all holes open, C#5 is produced. The normal range of the concert flute is from C4 to C7:



The pitches from C4 to Eb5 have their own individual fingerings:[12]



The pitches which are normally produced by overblowing the octave are the following:



C5, C#5, D5, Eb5, plus all the pitches of the flute range higher than those shown above <u>may</u> be produced by overblowing the octave, but they not always <u>are</u> produced in this way, because there are other ways of producing them which result in better tuning.

We recall now that $p' = 0$ at $x' = 0$, $x' = \frac{\lambda'}{2}$, $x' = (2)\frac{\lambda'}{2}$, $x' = (3)\frac{\lambda'}{2}$, $x' = (4)\frac{\lambda'}{2}$, $x' = (5)\frac{\lambda'}{2}$, $x' = (6)\frac{\lambda'}{2}$, etc... Thus the third mode of oscillation of the flute has a frequency of three times the fundamental, i. e. a twelfth above it. But this may also be regarded as $\frac{3}{2}(\lambda')$, where λ' is the previous mode. It is thus possible to produce pitches on the flute by overblowing at the fifth, although this is rarely done.

We continue now, in the same fashion, for three more modes of oscillation:

1) $x' = 2\lambda'$ = frequency 4 times the fundamental $= (\frac{4}{3})\frac{3\lambda'}{2}$ (where $\frac{3\lambda'}{2}$ is the previous mode.) This is the equivalent of overblowing by a perfect fourth (in the just scale).

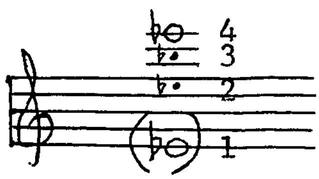
2) $x' = \frac{5\lambda'}{2}$ = frequency 5 times the fundamental $= \frac{5}{4}(2\lambda')$ (where $2\lambda'$ is the previous mode). This is the equivalent of overblowing by a major third (in the just scale).

3) $x' = \frac{6\lambda'}{2}$ = frequency 6 times the fundamental $= (\frac{6}{5})\frac{5\lambda'}{2}$ (where $\frac{5\lambda'}{2}$ is the previous mode). This is the equivalent of overblowing by a minor third (in the just scale).

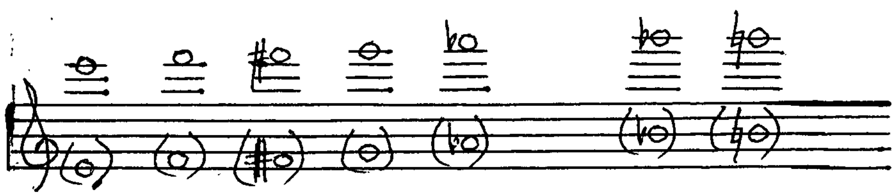
We may now consider the remainder of the pitches above C#6. The next pitch, D6, may be produced by overblowing D5, but it is more in tune when produced by overblowing G5 as the third partial:

(The numbers indicate relative frequencies of the pitches.)

The next pitch, Eb6, is produced by overblowing Eb4 at the double octave, as a fourth partial:



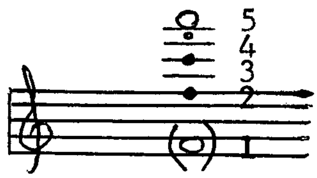
Beginning with the next pitch, E6, we are back to octave doublings of the first series of pitches shown. This method of pitch production remains consistent right up to the top of the range of the flute, except for the pitches A6 and C7:



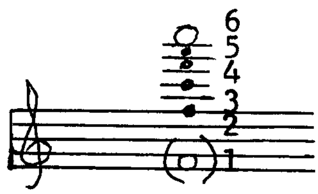
Missing: A6 C7

As was the case with Eb6, each of these tones is produced by overblowing at the double octave, i. e. each is the fourth partial of the note in brackets below it.

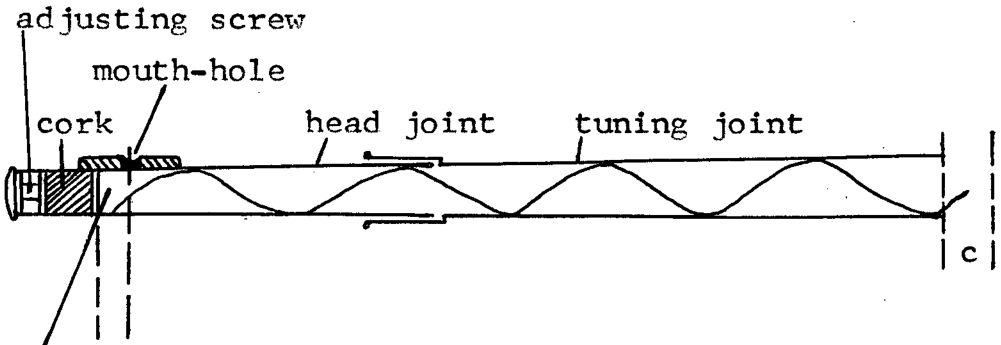
A6 is best produced as the fifth partial of F4:



C7 is -- surprisingly -- best produced as the sixth partial of F4:



There are several possible explanations for why these two pitches are best produced as shown above. Firstly, the opposite end of the flute may influence a pitch in this high a range. Although the full length of the flute is considered to be from the centre of the mouthhole to the remote tip of the flute, there is still molecular activity in the opposite (short) end on the other side of the mouthhole. Besides this, the total wavelength of the flute actually extends beyond the actual end of the tube, as shown in the following diagram.[13]



Cross-sectional view of the essential parts of a flute (not drawn to scale).

The distance marked "c" is known as the end correction, and the effect of this end correction is the second possible explanation for producing A6 and C7 as shown above.

Testing the Equation $v = f\lambda$[14]

As was done with the clarinet, we now test the equation $v = f\lambda$ in a practical situation involving the flute. With a flute of length L, $L = \frac{n\lambda}{2}$ must be satisfied, therefore $\lambda_{max} = 2L$ (see page 19). The total length of the flute (from the mouth-hole to the open end) is 60 cm, therefore $\lambda_{max} = \lambda$ for the lowest pitch of the flute = 120 cm. With

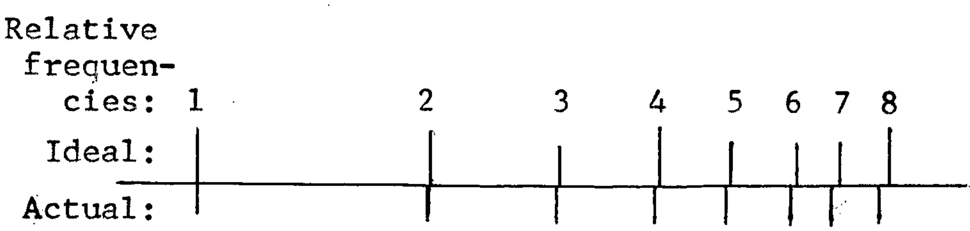
v = 33853 cm/sec, substituting into $v = f\lambda$, we get:
33853 cm/sec = f(120 cm) $\therefore$ f = 282.1 hz.

The closest frequency to this in the equal-tempered scale is that of C#4, whose frequency is 277.18 hz. In reality, the lowest pitch of the flute is C4, a semitone lower. For further comparison, three more similar calculations were made. In each case, the length L was measured from the centre of the mouth-hole to the centre of the first open hole. The results are summarized in the table on the following page.

The explanation for the discrepancies in the table is that the flute actually differs in a number of ways from a simple tube open at both ends. For all but the lowest note on the instrument, one end of the vibrating air column is actually an open side hole on the tube. The key pad for this hole is close enough to have an effect on the air column vibrations, and the remainder of the tube beyond the open hole also has an influence. The actual arrangement is, therefore, more complicated acoustically than a simple open end. Also, the other end of the air column is the mouth-hole,

1a	Examined Pitch:	C4	D4	F4	C#5
1b	Known frequency of this pitch (hz):	261.63	293.66	349.23	554.37
	Length of pipe (L) (cm):	60.0	52.0	43.0	23.6
	λ (where $\lambda_{max} = 2L$) (cm):	120.0	104.0	86.0	47.2
	Calculated frequency (33853 cm/sec $\div$ 2L) (hz):	281.1	325.5	393.6	712.2
2a	Pitch whose frequency in equal temperament is nearest to that obtained above:	C#4	E4	G4	F5
	Known frequency of this pitch (hz):	277.18	329.63	392.00	698.46
	Discrepancy of 2a with respect to 1a (by interval):	- 2nd hghr.	+ 2nd hghr.	+ 2nd hghr.	+ 3rd hghr.
	Ratio of calculated frequency with respect to 1b:	1.07	1.11	1.13	1.29

which is more or less covered by the lip. In the portion of the flute between these two ends are a number of side holes covered with key pads; these add an additional volume in this region as compared to the plain tube. Furthermore, the head joint of the flute is not cylindrical at all, but is, in fact, conical, narrowing down to a diameter of 1.7 cm from the 1.9 cm diameter of the remainder of the flute. Thus, although a pipe 60 cm in length open at both ends should sound C#4 as its lowest pitch, in reality, C4 is the lowest pitch of the flute.[15] As is evident from the table, the the higher the pitch, the greater the discrepancy between the calculated frequency and pitch with respect to the examined frequency and pitch. This exemplifies what is known as the "shifting of overtones". Although ideally, harmonics should not be out of tune, in the equal-tempered scale, such an occurance as the following is not infrequent:

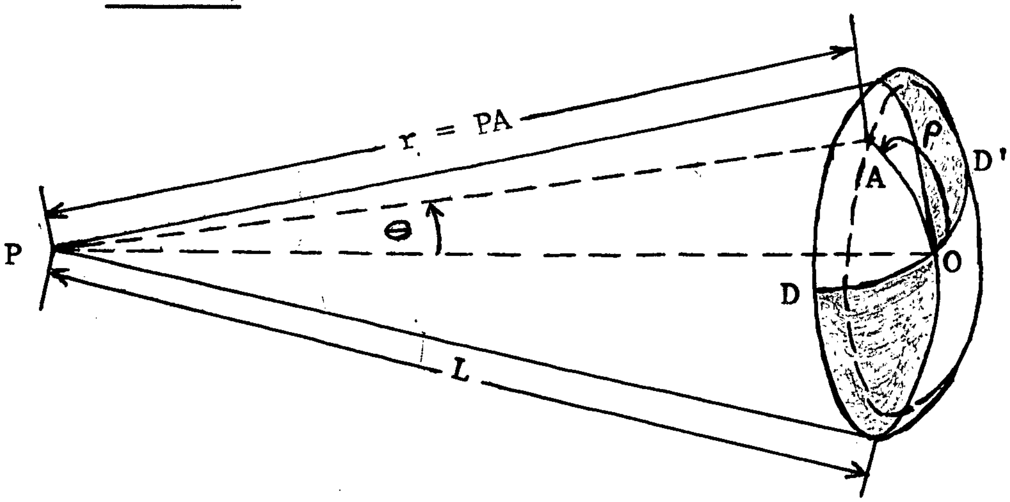


In the ideal case, overblowing follows the logarithmic scale precisely; in the actual case, the higher the harmonic, the flatter it becomes in comparison to the corresponding ideal pitch. Thus, by the time the highest register of the flute is reached, overblowing may not necessarily produce the pitch it is "supposed to" in tune, and it may well be possible to produce the desired pitch in tune through a more distantly related method of overblowing. This brings us back to the pitches A6 and C7 discussed previously. These two pitches serve as illustrative examples of the situation just described.

III. THE CONICAL TUBE [1]

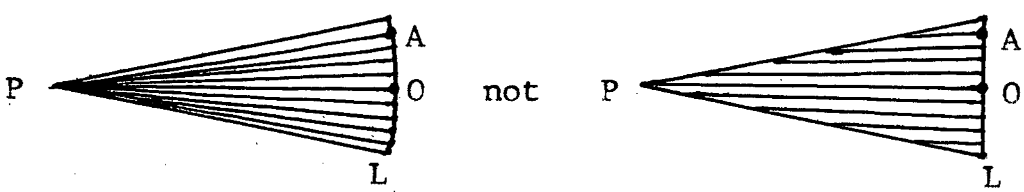
Two cases of the conical tube will be considered: 1) full tube with peak, 2) the same tube with its peak cut off.

Case 1:



For large r and L, the lengths of r and L are taken to be equal.

The diagram above is not a cone according to the precise definition of the word, because the open end is not flat, but spherical. This is because waves in a cone travel radially and not in parallel lines at right angles to the base of the cone:



Consequently, the distance PL is precisely equal to the length of the axis $\overline{PO}$ and to the distance r = PA. Clearly, if the situation were as shown in the second of the two diagrams, the distance PL would be slightly larger than $\overline{PA}$, although not by much. Another consequence of having this type of "base" for the cone is that this "base" is not an ordinary circle with diameter $\overline{DOD'}$, because there is curvature involved, although it is very slight, especially for small $\overline{DOD'}$. For comparison, the cone of case 2 is drawn with a flat base. The three-dimensional co-ordinate system, called the spherical polar system, used here operates thus: A point "A" in the cone is defined by specifying r, θ, and ρ, where r = $\overline{PA}$, θ is the angle formed between the axis $\overline{PO}$ and $\overline{PA}$, and ρ is the angle formed between $\overline{OA}$ and $\overline{DOD'}$ (where $\overline{DOD'}$ is the "diameter" of "circle" O).

The wave equation in this case is a generalisation to three dimensions; thus it involves three variables (x, y, z) with $x = r\sin\theta\cos\rho$, $y = r\sin\theta\sin\rho$, and $z = r\cos\theta$. Now the general wave equation is:

$$\nabla^2 p = \frac{1}{c^2} \cdot \frac{d^2 p}{dt^2} \qquad (1)$$

In cartesian co-ordinates, that is, in terms of x, y, and z,

$$\nabla^2 p = \frac{d^2 p}{dx^2} + \frac{d^2 p}{dy^2} + \frac{d^2 p}{dz^2} \,. \qquad (2)$$

In this equation, a set of three numbers, x, y, and z, defines a point which can also be expressed in terms of r, Θ , and ρ . In this case, when we transform the expression in x, y, and z to that in r, Θ, ρ , we find that each of the last two terms becomes equal to zero, because there is no variation involved with either y or z (i. e. with either Θ or ρ), since θ represents radial waves in cross section and ρ represents circular waves. Now from equation (2) above, through the transformation from cartesian co-ordinates to polar co-ordinates, we get:

$$\nabla^2 p = \frac{1}{r^2} \frac{d}{dr}\left(r^2 \frac{dp}{dr}\right) + \left\{ \text{a term in } \Theta + \text{a term in } \rho \right\}$$

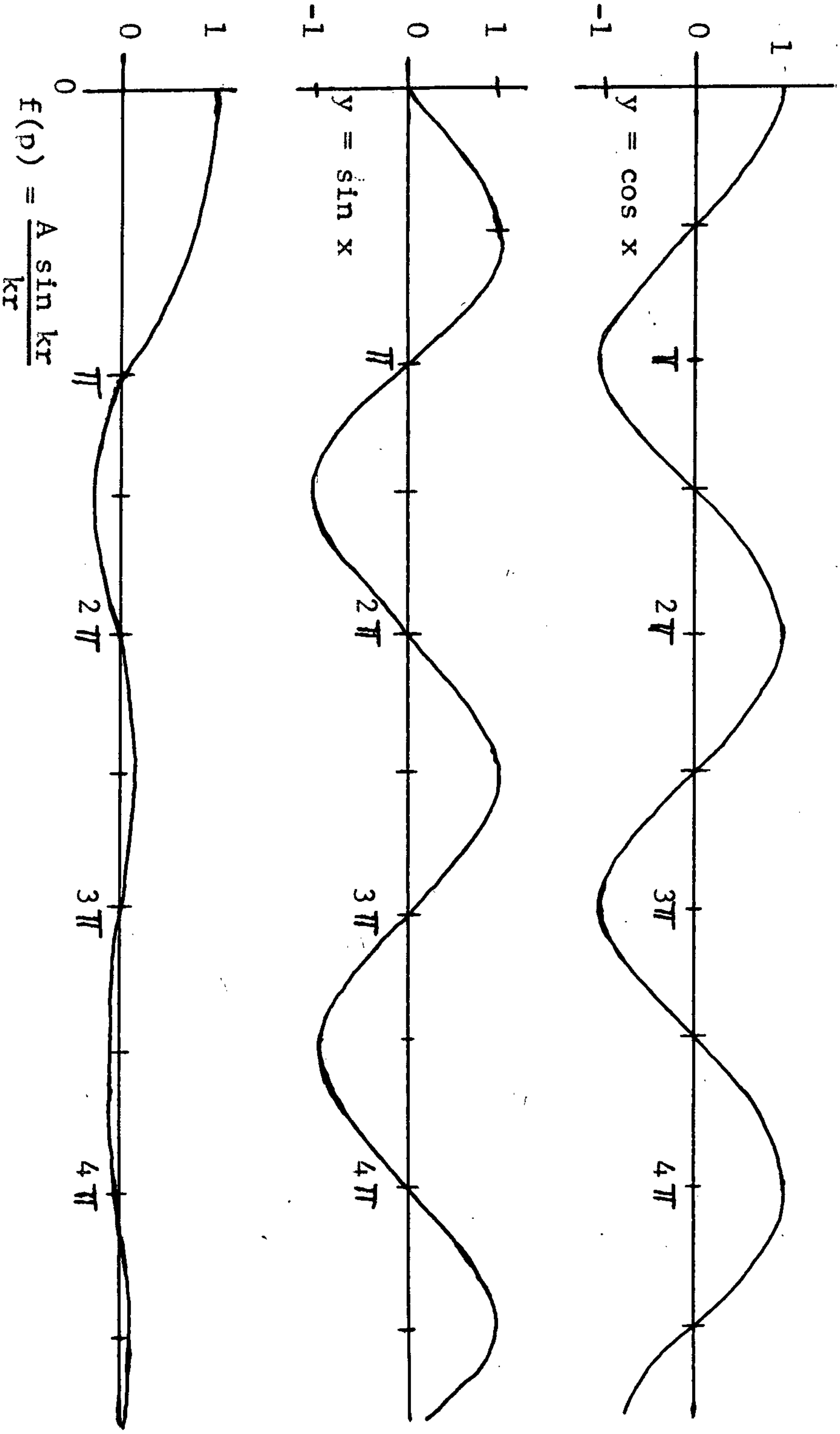
$$= \frac{1}{c^2} \frac{d^2 p}{dt^2} \qquad (3)$$

The bracketed terms in θ and ρ are omitted, because variations with θ and ρ will be associated with small distances (of the order of the diameter), and hence with high frequencies. Consequently, they will not be taken into consideration. The general solution for equation (3) in this case is:

$$p = \frac{A \sin kr}{kr} + \frac{B \cos kr}{kr} \text{ (at any instant).} \qquad (4)$$

The term $\frac{B \cos kr}{kr}$ is omitted, because it goes to infinity at zero; therefore only $\frac{A \sin kr}{kr}$ remains. This term remains finite, because for small x, sin x = x. The graph of this equation is the third graph on the following page. The cosine and sine graphs are given first for purposes of comparison. The following observations may now be made:

1) The graph of equation (4) is a type of combination of the cosine and sine graphs. It resembles the cosine graph in that it reaches its maximum amplitude of 1 at 0. This means, that in practical application with an instrument, there will be a pressure antinode at the mouthpiece, as was the case with the clarinet. It resembles the sine graph much more than it does the cosine graph, because it crosses zero on the x-axis at $n\pi$ for any integer n (except for n = 0). Furthermore,



y = cos x
y = sin x
f(p) = A sin kr / kr
1
0
-1
π
2π
3π
4π

this curve ascends and descends comparatively with the sine curve. All this means that in practical application with an instrument, the modes of oscillation will be (both even and odd) integral multiples of the fundamental. Consequently, overblowing will occur at the octave (as in the case of the flute), and not at the twelfth (as in the case of the clarinet).

2) The amplitude a of the graph of equation (4) decreases as x increases:

$$\text{at } \frac{3\pi}{2},\ a = \frac{2A}{3\pi}$$

$$\text{at } \frac{5\pi}{2},\ a = \frac{2A}{5\pi}$$

$$\text{at } \frac{7\pi}{2},\ a = \frac{2A}{7\pi}$$

$$\text{at } \frac{9\pi}{2},\ a = \frac{2A}{9\pi}\ ,\ \text{etc...}$$

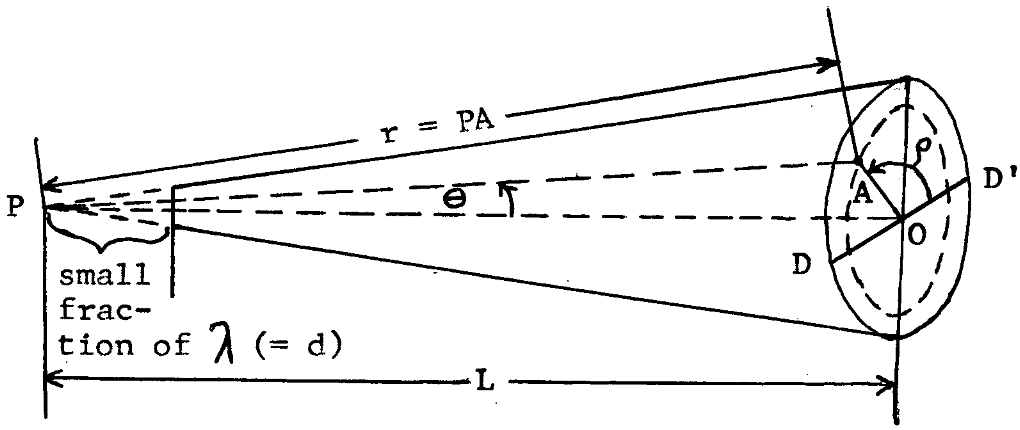
This is because the flow of energy through a conical tube follows the inverse square law:

$$E \propto a^2 = \left(\frac{A}{kr}\right)^2 , \qquad (5)$$

$$\therefore\ E \propto \frac{1}{r^2}\ .$$

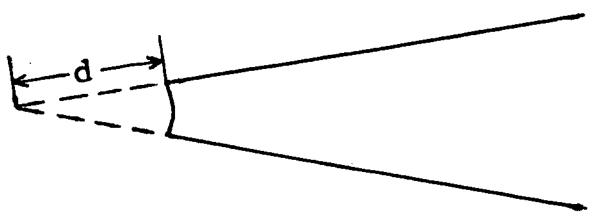
<u>Conclusion:</u> The closed conical tube has the same resonant frequencies as the cylindrical pipe open at both ends.

Case 2:



For large r and L, the lengths of r and L are taken to be equal.

For purposes of calculation, P remains in the same position as in the full conical tube. A cone with its peak cut off behaves almost exactly as if it were a full cone, because the distance designated "d" in the diagram is a very small fraction of the entire wavelength of the tube. Furthermore, the volume of the truncated portion of the cone is an even smaller fraction of the volume of the entire cone. In the case which we are considering here, we have a closed end at r = d:



Getting back now to equation (4) (as on page 51):

$$p = \frac{A \sin kr}{kr} + \frac{B \cos kr}{kr} . \qquad (4)$$

For convenience in differentiation, we may change this to the equivalent expression:

$$p = \frac{C \sin (kr + \varepsilon)}{kr} \qquad (5)$$

$$\frac{\partial p}{\partial r} = - \frac{C \sin (kr + \varepsilon)}{kr^2} + \frac{k\, C \cos (kr + \varepsilon)}{kr} . \qquad (6)$$

Now at the closed end when r = d we require that $\frac{\partial p}{\partial r} = 0$, i. e. that the right-hand side of (6) = 0.

$\therefore - \sin (kd + \varepsilon) + kd \cos (kd + \varepsilon) = 0.$

This condition does not have a simply expressed general solution, but if kd and ε are both small (i. e. if their squares are much smaller than one), then we can make an approximation as follows: For small x, (i. e. as x approaches zero), sin x approaches x and cos x approaches 1. Thus for small kd and ε the condition becomes:

$$- (kd + \varepsilon) + kd \cdot 1 = 0$$

$$\therefore \ \varepsilon = 0, \qquad p = \frac{C \sin kr}{kr} .$$

Thus the pressure equation is identical to that for the full cone (see page 51). The previous analysis, therefore, can be used. The above is, in essence, a mathematical consequence of the flatness of the curve of the third graph on page 52 when it is near zero.

Boundary Conditions for the Conical Tube

As was the case with the cylindrical tube, there are theoretically four possible boundary conditions for the conical tube:

1) tube open at end one, closed at end two,
2) tube open at end two, closed at end one,
3) tube closed at both ends,
4) tube open at both ends.

This time, from the musician's point of view, only case 1 is of interest, since it represents the bore of the oboe (and the english horn) and the bassoon. Case 2 equals case 1; case 3 has no application, because no sound is emitted from a tube closed at both ends and without any side holes. Case 4 has no musical application either, because there is nothing to be gained from constructing an instrument with a conical tube open at both ends, since the cylindrical tube open at both ends has the same properties as the closed conical tube, as was shown in the previous section. Although it was also shown in the previous section that the pressure expression for the truncated cone is identical to that for the full cone, it should be specified here that the bores of the oboe and bassoon are examples of the truncated cone. We start, now, with the general equations for <u>pressure</u> p and <u>velocity</u> v:

$$p = \frac{A \sin kr}{kr} + \frac{B \cos kr}{kr} \qquad (1)$$

$$v = \frac{\partial p}{\partial r} =$$

$$-\frac{A \sin kr}{kr^2} + \frac{kA \cos kr}{kr} - \frac{kB \sin kr}{kr} - \frac{B \cos kr}{kr^2} \qquad (2)$$

Now for a tube closed at one end, we require, as before, that $v = \frac{\partial p}{\partial r} = 0$. Thus, after putting equation (2) equal to zero, we may multiply through by $-kr^2$ to get: $v = \frac{\partial p}{\partial r} = A \sin kr - kr\, A \cos kr +$

$$+ kr\, B \sin kr + B \cos kr = 0. \qquad (3)$$

To impose boundary conditions, we first put r = 0 in equation (3) above, thus ending up with:

$$v = \frac{\partial p}{\partial r} = B \cos kr = 0, \therefore B = 0.$$

Now equation (1) becomes:

$$p = \frac{A \sin kr}{kr} \qquad (4)$$

We next put $r = L$, where L represents the other (open) end of the pipe. Equation (4) now becomes:

$$p = \frac{A \sin kL}{kL} .$$

We know that at the open end of the tube, $p \doteq 0$,

$$\therefore \frac{A \sin kL}{kL} = 0, \therefore kL = m\pi , \therefore$$

$$k = \frac{m\pi}{L} \text{ for any integer } m, \text{ excluding } m = 0, \qquad (5)$$

because this would imply division by zero above. This is the value of k which is fixed by the boundary conditions. On the other hand, we expect a simple harmonic wave to be in the form:

$$p = B\cos\frac{2\pi}{\lambda}x. \qquad (6)$$

Therefore by definition in terms of wavelength,

$$k = \frac{2\pi}{\lambda} . \qquad (7)$$

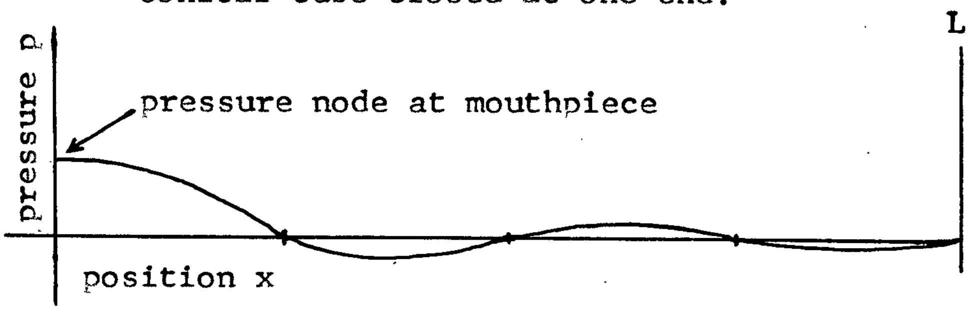
Now, combining equations (5) and (7) we obtain:

$$\frac{m\pi}{L} = \frac{2\pi}{\lambda} , \therefore$$

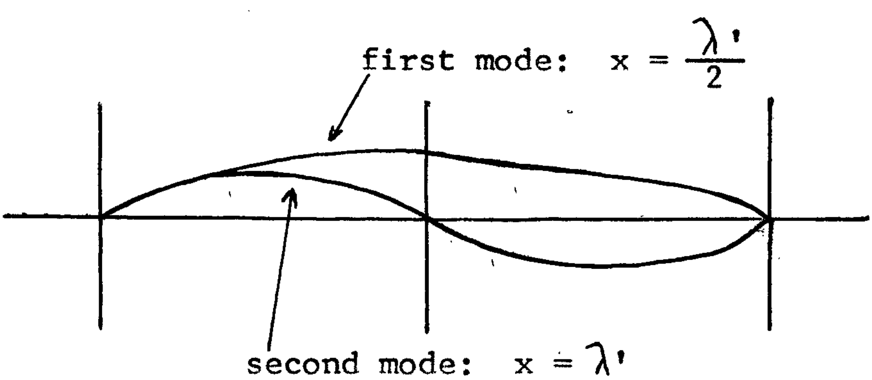
$$L = \frac{m\lambda}{2} \text{ for any non-zero integer } m.$$

Now we may find all values of x for which $p = 0$ by finding all values of x for which $\frac{2\pi}{\lambda}x = m\pi$ for any non-zero integer m. Therefore $x = \frac{m\lambda}{2}$ for any integer m, m $\neq$ 0. This means that p = 0 when $x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda, \frac{5\lambda}{2}, 3\lambda, \frac{7\lambda}{2}, 4\lambda,$ etc... The general equation involving λ of the conical tube of length L closed at one end (the oboe or the bassoon) is $L = \frac{m\lambda}{2}$ for some non-zero integer m. In the diagram below, m is assigned the value of 4.

Conical tube closed at one end:



Since $L = \frac{m\lambda}{2}$ must be satisfied, $\lambda_{max} = 2L$. As stated above, here $p = \frac{A \sin 2\pi r/\lambda}{2\pi r/\lambda}$. Consequently, p = 0 beginning at π every 180°.



Possible Frequencies

The Oboe

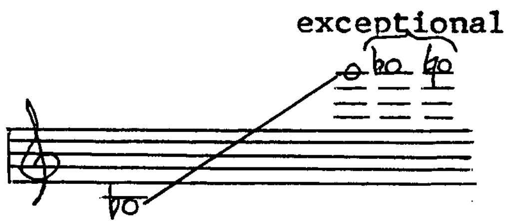
The oboe is basically a tube with a conical air column in which the tip of the cone has been cut off and a double reed attached. The double reed consists of two halves of cane beating against each other. The oboe is constructed of three pieces of wood: the top joint, the bottom joint, and the bell. All three sections carry keys; this is in direct contrast to the clarinet, whose bell carries no keys. The oboe reed is attached to a conical piece of metal tubing called a staple, which is inserted into the top joint.[2]

Since the conical tube closed at one end vibrates with modes that are integral multiples of the fundamental, the second mode of oscillation of the oboe has a frequency twice the fundamental:

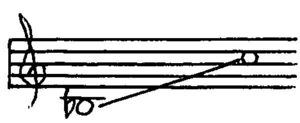
$p = 0$ first at $x = \frac{\lambda}{2}$; next at $x = \lambda$.[3]

Musically, this results in a rise in pitch of an octave, which is the basic interval by which the oboe overblows. This property, plus the fact that the oboe has a pressure node (and not an antinode) at the mouthpiece, makes the construction and fingerings of the oboe relatively simple, i. e. similar to that of the flute and in contrast to that of the clarinet. The oboe has 10 to 16 "additional" holes and keys, which is considerably less than the clarinet, but is more than the flute.

The basic scale of the oboe is the six-hole, seven-note scale, which starts on D4 with the six holes closed. Additional holes and keys on the lower joint and bell are used to provide the sharps and flats and to extend the range down to Bb3. The normal range of the oboe, then, is the following:

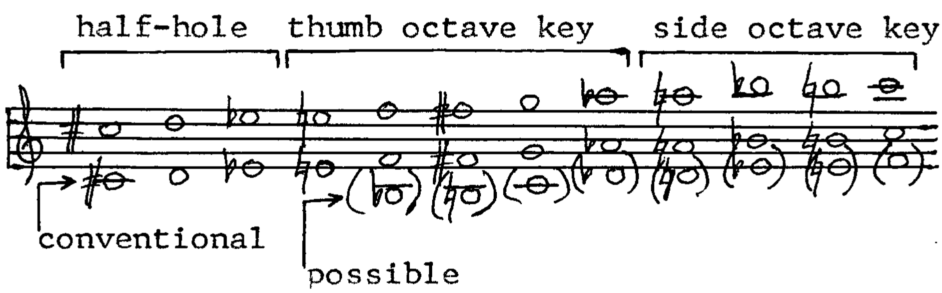


The pitches from Bb3 to C5 have their own individual fingerings:[4]



The pitches Bb4, B4, and C5 <u>may</u> be produced by octave overblowing, but this is not done, because it is very difficult and because the tone quality suffers greatly. The Bb5 turns out sharp when produced in this way, but the B5 and C5 are fairly well in tune.

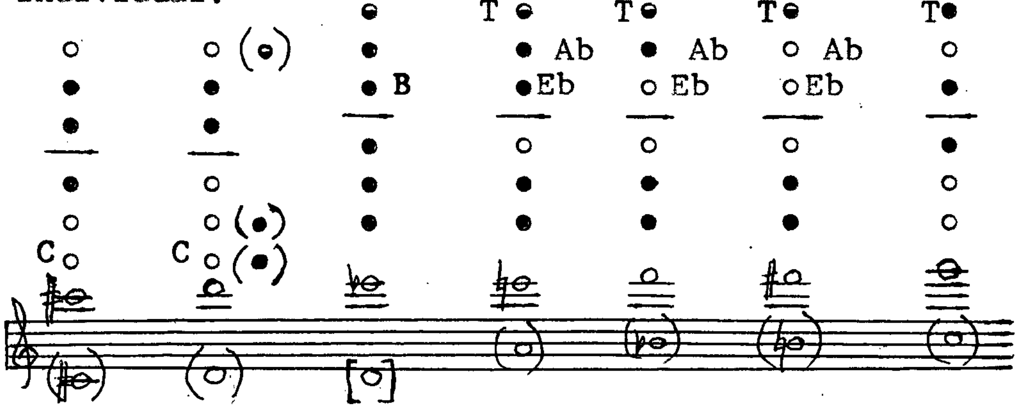
Between C5 and C#5 is where the first "break", i. e. the first change in register, occurs. The pitches which <u>are</u> normally produced by overblowing the octave are the following:



In each case, the fingering is identical to that used to produce the pitch an octave lower, except that one of the three octave keys is added, as indicated above.

We recall now, that p = 0 at $x = \frac{\lambda}{2}$, $x = \lambda$, $x = \frac{3\lambda}{2}$, $x = 2\lambda$, etc... This means that the third mode of oscillation of the oboe has a frequency of three times the fundamental, i. e. a twelfth above it. Of the pitches shown above, from F5 upwards, the pitches shown may be obtained by overblowing the twelfth, but this is not often done. If they are produced in this way, these tones take on a peculiar character of their own. These special harmonics are a useful addition to the oboist's resources, particularly in solo work, but they require care in production and cannot be played forte. When used with discretion, their very individual tone colour can be most effective. It should be added here that the oboe is undoubtedly at its best in this (medium) compass, i. e. approximately from F4 to B5 (varying with different models), since in this range its tone is at its sweetest, neither too reedy as it sometimes tends to be lower down, nor too thin, as it may be above B5.

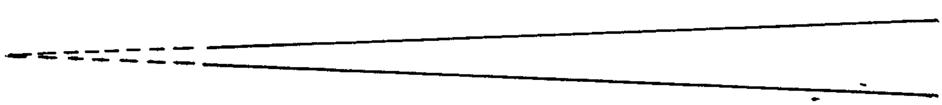
Between C6 and C#6 is the point at which the next break of the oboe occurs. From the practical standpoint, the fingerings for the seven highest pitches conventionally played on the oboe are quite individual:



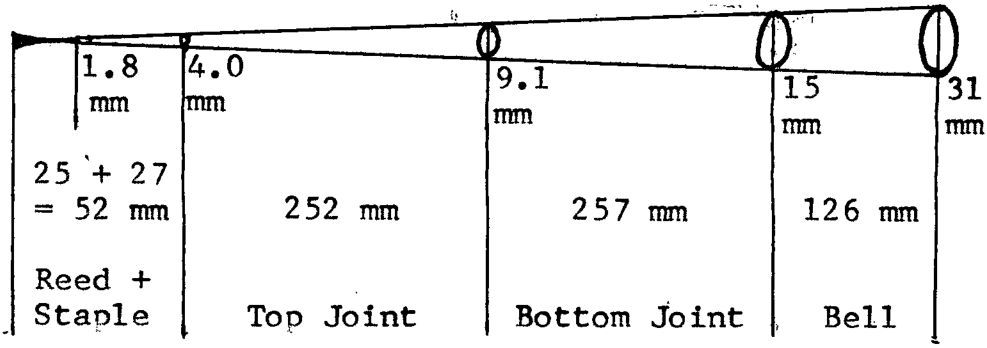
From the acoustical standpoint, however, we find that it is possible to produce these tones either as twelfths (i. e. third harmonics) or as super-octaves (i. e. fourth harmonics.) In the case of the super-octaves, this means that the oboe is in its fourth mode of oscillation, in which the frequency is four times the fundamental (2λ as compared to $\frac{\lambda}{2}$.) The two notes D6 and Eb6 require additional discussion. To finger D6, the oboist may, at will, choose to close the holes indicated in brackets. To finger Eb6, these holes must be closed, and the C key must be replaced by the B key. Since these two notes may thus be fingered so similarly, it is possible to think of Eb6 as being produced (from the acoustical and <u>not</u> from the practical standpoint) as an overblown D4 at the double octave which is tuned to Eb6 by use of the B key. We must remember, that in this high a range, overblowing rarely results in pitches which are in tune in the equal-tempered system. That is why the fingerings shown above are the ones that are, in fact, used -- they result in better tunings than those achieved by overblowing. When the player has mastered both the conventional and the non-conventional fingerings, he will soon notice minute differences between them both in quality and in intonation. He may even discover additional or modified fingerings which suit his own instrument particularly well. The oboist should try to take advantage of such alternatives, as much as his abilities and musicianship allow him to, because this is all part of the technique of oboe-playing. Such things transform a competent performance into an artistic one and greatly increase the pleasure of both the player and his audience.[5]

Testing the Equation $v = f\lambda$[6]

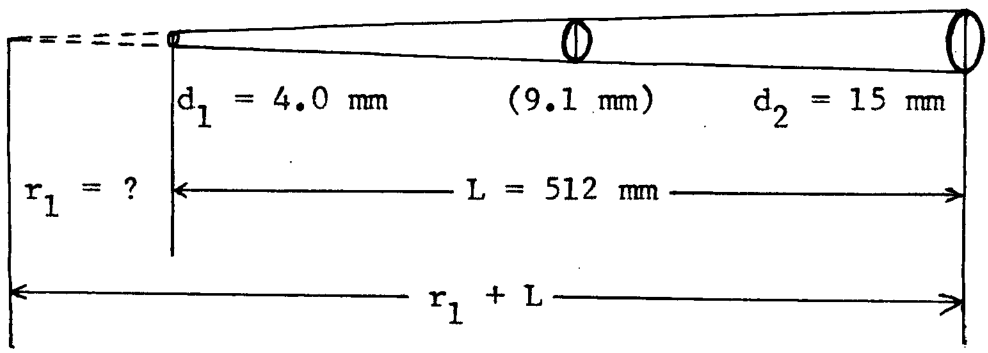
We now want to test the equation $v = f\lambda$ in a practical situation involving the oboe. With an oboe of length L, $L = \frac{n\lambda}{2}$ must be satisfied, therefore $\lambda_{max} = 2L$ (see page 59). We must first, however, calculate the theoretical length of the conical tube of the oboe by extrapolating the conical bore to the imaginary peak:



Now the measured dimensions (diameters and lengths) of the parts of the oboe are as follows:



Since the Reed + Staple combination and the bell are the places at which the oboe tube deviates from its basically conical shape, we do not consider these in the extrapolation, and make use of only the top and bottom joints:



With $d_1 = K(r_1)$, let $d_1 = 4$ and $d_2 = K(r_1 + L) = 15$.

Then $KL = d_2 - d_1 = 11$, $\therefore$ $K = \frac{d_2 - d_1}{L} = \frac{11}{512} = .0215$.

Now $r_1 + L = \frac{d_2}{K} = \frac{15}{.0215} = 698$, $\therefore r_1 = 186$ mm.

To verify the value obtained for K, we can take the joints separately, as follows:

Top Joint:

let $d_1 = K(r_1) = 4$ and

$d_2 = K(r_1 + 252) = 9.1$

Then $K(252) = 5.1$

$\therefore K = \frac{5.1}{252} = .0202$

Bottom Joint:

let $d_1 = K(r_1) = 9.1$ and

$d_2 = K(r_1 + 257) = 15$

Then $K(257) = 5.9$

$\therefore K = \frac{5.9}{257} = .0230$

Thus K averages out to be $\frac{.0202 + .0230}{2} = .0216$ by this method as compared to .0215 by the method used above. For the four pitches whose corresponding wavelength was measured, the measurement was taken to the end of the top joint (i. e. to the point at which the diameter of the tube is 4.0 mm) as shown in line "a" below. When r_1 is added to each of these lengths, we get the results in line "b", which are then taken to be the length L in the table.

Pitch:	Bb3	B3	F#4	C5
a) Distance from top of top joint to first open hole (cm):	52.0	46.5	27.5	15.2
r_1 (cm):	18.6	18.6	18.6	18.6
b) L (cm):	70.6	65.1	46.1	33.8

We recall now, that the speed of sound at body temperature is approximately 33853 cm/sec. This is the last value which we need in order to use the equation $v = f\lambda$ and to complete the table.

1a	Examined Pitch:	Bb3	B3	F#4	C5
1b	Known frequency of this pitch (hz):	233.08	246.94	369.99	523.25
	Length of pipe (L) (cm):	70.6	65.1	46.1	33.8
	λ (where $\lambda_{max} = 2L$) (cm):	141	130	92	67.6
	Calculated frequency (33853 cm/sec $\div$ 2L) (hz):	240.0	260.4	368.0	500.8
2a	Pitch whose frequency in equal temperament is nearest to that obtained above:	Bb3	C4	F#4	B4
	Known frequency of this pitch (hz):	233.08	261.63	369.99	493.88
	Discrepancy of 2a with respect to 1a (by interval):	none	- 2nd hghr.	none	- 2nd lower
	Ratio of calculated frequency with respect to 1b:	1.03	1.05	.995	.957

Conclusion: When we substitute for r and for the pitch C5 in $Kr = \frac{2\pi r}{\lambda}$, we obtain $\frac{2\pi \times 186}{338(2)}$, which = 1.73, which is not negligible in comparison to one. Since the results above come so close to the ones desired, we must conclude that for small changes from the ideal conical shape of the tube, we can work with equivalent volumes whether the lengths involved are equivalent or not. (The volume of the reed + staple combination is approximately equal to that of the truncated portion of the cone, which is approximately three times longer.)

Other Aspects of the Acoustics of the Oboe in Particular and Wind Instruments in General[7]

As is the case with all wind instruments, the oboe has a generator or exciter and a resonator. In the oboe, the generator is the double reed, which may be regarded as a sort of valve which transforms a steady stream of air from the player's lungs into a pulsating one. This feeds periodic bursts of energy to the main air mass in the tube of the instrument. Next comes the resonator, i. e. the tube or body of the instrument, or, better still, the column of air contained within that body. Taken together, the generator and the resonator form a paired dynamic system, and the vibrations of the two are closely associated. The generator and resonator should not be considered separately, because then one tends to neglect such details as the fact that in the oboe the mass of air in the staple and between the hollow reed-blades constitutes in itself a considerable proportion of the air column. In fact, under playing conditions, the generator and resonator greatly influence each other's behavior and one cannot arrive at proper conclusions by examining either in isolation. Finally, there is one more common feature which can be noticed only during the act of playing. Then the generator is coupled both to the resonator and to the air cavities of the head, throat, and chest, and the air in them is also set in vibration. Consequently, during performance, the oboe-player and his instrument together form a coupled system of <u>three</u> elements. In playing, the oboist can vary the length of the resonator not only with his fingers, but also by adjusting the reed with his lips, pinching or relaxing as required, and by modifying the shape and volume of his air cavities to some small extent by the use of the chest and throat muscles. Here exactly lies the source of that flexible intonation so highly valued by musicians.

As was concluded earlier, the truncated conical tube closed at one end resonates to the entire series of harmonic overtones, i. e. the musical octave, twelfth, fifteenth, etc... Unfortunately, musical instruments rarely coincide directly with theoretical calculations (and conclusions). These take no account of: 1) the way in which the sound waves have been excited, 2) the changes in magnitude, range, or pitch of overtones due to the coupling of the generator with the resonator, 3) altered frequencies due to the need for sideholes, and 4) many other features which practical use or construction impose on the instrument. Let us take, for example, one feature which seems fairly obvious at sight. We know that our calculations were greatly simplified by extending the bore of the oboe to its theoretical peak. But such an extension cannot be made in practice, and the reed + staple combination must substitute for the missing part as best it can. Until quite recently, this has been done without much understanding of the dimensions involved beyond that gathered from practical experience, trial, and error. Recently, however, it has been shown that if the internal volume of the reed + staple combination slightly exceeds that of the missing tip of the cone, a suitable interplay with other features of the rest of the air column (which is in itself a modified cone) can be brought about, and the result is a fairly good acoustic equivalent with musically useful ratios between the different resonance frequencies.

If we listen to an ascending scale played through the compass of the oboe, we distinguish changes in quality between successive notes and a progressive thinning of tone, although the general impression is one of homogeneity. Generally, the more interesting (although not always the most pleasing or aesthetically satisfying) tones to the ear are those containing many powerful overtones. These are more prominent in loud sounds than in soft. The sound radiated by an instrument also generally gets less complex in the higher registers. In general, each successive note produced on the oboe does <u>not</u> reproduce the same relative harmonic pattern simply based on a new fundamental as one hole after another is opened.

As has already been mentioned, in spite of the audible differences between adjacent notes, a scale played on the oboe leaves the ear with an impression of homogeneity throughout the entire range. This leads one to suspect that there is really some common factor running through the entire compass of the oboe, and that so far this factor has not yet been identified. At one time, (around 1837) it was thought that the theory of <u>formants</u> provided the answer to this question. According to this theory, set forth by Charles Wheatstone and Hermann Helmholtz, when a vowel is sounded by any human voice, regardless on which note it is sounded, it is identified by certain vibration frequencies which are invariably present. This phenomenon is conveniently described by saying that the air-cavities of the head possess qualities of <u>selective resonance</u>. Around 1897, the physicist Harmann-Goldap (who was the one to actually invent the word "formant" to describe the Wheatstone-Helmholz theory), assumed an analogy of the human voice to orchestral instruments, and showed a characteristic array of frequencies present throughout the compass. He, therefore, claimed that orchestral instruments, like the voice, possessed formants, and that this was the explanation for instrumental timbre. The idea of the formant frequency also gains support from experiments done with the cor anglais. The data gathered are as follows:

1) A typical cor anglais bell shows as its chief characteristic a pronounced resonance in the region of 680 cycles per second.

2) Many people have confirmed the impression that this instrument in its lowest octave sings "aw". The characteristic frequency of this sound has been found to be 730 cycles.

3) It has been separately concluded, that each vowel shows two ranges of characteristic frequencies, a higher and a lower.

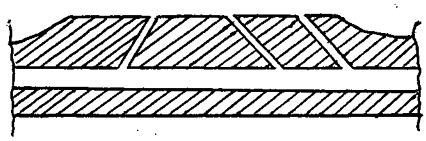
4) The nearest frequency in a recognized speech sound to 680 cycles is the lower one (704), typical

of the shortened ŏ as in the word "hot".

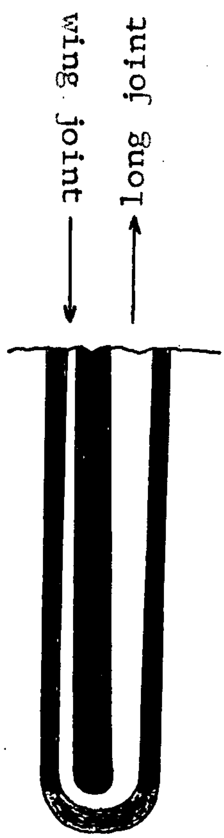
Since the time of Hermann-Goldap, research has shown that while selective resonances are involved in both vocal and instrumental sound production, their sources are not comparable in general terms. The source of instrumental timbre is now known to lie in the shape and dimensions of the bore and its array of note-holes, plus the effect of various properties of the reed.

The Bassoon

The bassoon is essentially a conical tube with a total length of about 250 cm. Since this would be too long an instrument if left straight, it is folded at a 180° angle, to bring it down to a manageable length (about 122 cm.) The narrow end of the bassoon cone consists of a piece of tapered metal tubing called the crook or bocal, which has a diameter of approximately 4 mm at the point where the double reed is attached. The crook bends first upward and then down, and is inserted into the wing joint. This joint (made of wood) has the top three holes of the basic scale, which are covered by the three fingers of the left hand. These holes, if bored straight through at the proper places, would be too far apart to be covered by the fingers of one hand. Consequently, the wood is thickened in this region, forming the "wing", and the holes are bored at a slant, as shown below:



This type of construction allows the holes to be far enough along the bore as well as close enough together to be covered by the fingers. The wing joint is inserted into a section called the <u>boot</u>, also wooden, which contains two holes, one of these being the continuation of the bore of the wing joint. The boot carries the bottom three tone holes of the basic scale, for the right hand. A metal piece formed into a U-shaped tube is clamped to the bottom of the boot to connect its two conical holes so as to produce a continuous bore:



Into the boot is fitted the long joint, which carries the conical bore up to the bell; this is the piece carrying the characteristic metal or ivory ring seen on the top of the bassoon. At the end of the bell the diameter of the conical bore is about 40 mm.[8]

At this point, a comparison may be made between the bassoon and the traditional pipe organ. In the organ, each note of the scale has its own separate pipe, so that in a large organ several thousand pipes may be required, each of which will have its own generator and resonator. Thus it becomes necessary to supply air to the pipes from a common source, and the wind pressure remains constant for each pipe.

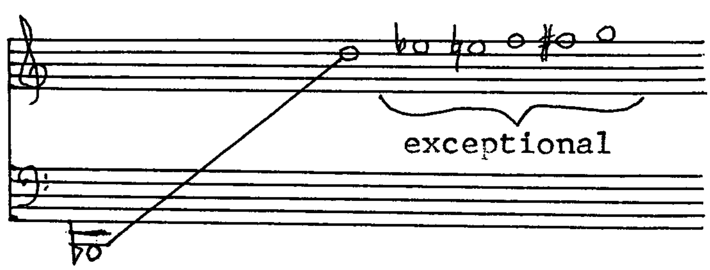
With the bassoon, the case is entirely different. The player must acquire artistic virtuosity and adaptability to the numerous requirements of breath-control and skill of the fingers. Bassoon playing requires complete mastery of a chromatic sequence in equal temperament over 3½ octaves. In other words, about 44 notes must be produced from a single wooden pipe, and to do so, 8 fingers and 2 thumbs must control 18 tone or semitone holes as well as 3 harmonic holes.[9]

We recall now, that the conical tube closed at one end vibrates with modes that are integral multiples of the fundamental. Thus, the second mode of oscillation of the bassoon has a frequency twice the fundamental:

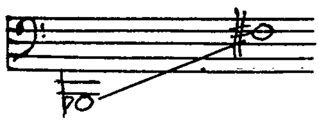
$p = 0$ first at $x = \frac{\lambda}{2}$; next at $x = \lambda$.[10]

Musically, this results in a rise in pitch of an octave, which is the basic interval by which the bassoon overblows.

The basic scale of the bassoon is the six-hole, seven-note scale, which starts on G2 with the six holes closed. For the nine chromatic notes below G2, all six holes, plus additional holes operated by keys, must be closed. The normal range of the bassoon, then, is the following:

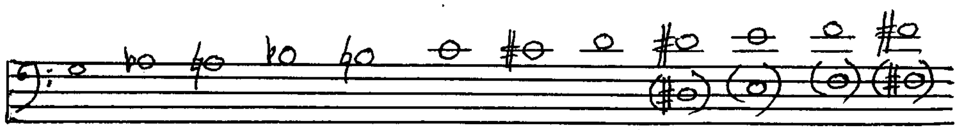


The pitches from Bb1 to F#3 have their own individual fingerings: [11]



With the basic scale being G2, A2, B2, C3, D3, E3, F3, the fingering for the "black notes" in between is always that of the previous "white note" plus an additional hole or key.

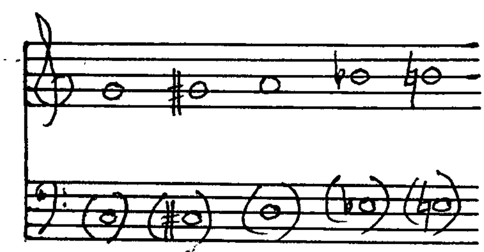
Overblowing at the octave begins with the next note, G3. From G3 to D4, this is the conventional method of pitch production.



From D#4 to F#4 overblowing at the octave is possible, but there are other "artificial" fingerings which result in better-tuned pitches. For overblowing at the octave, fingering is identical to that used to produce the pitch an octave lower, except that the whisper key (which was used before) is not used. Furthermore, vent-holes are used. Since, however, the bassoon is so large, one vent-hole does not suffice, as it does on the clarinet.

The vent-hole must move up the instrument as the scale is ascended. This is accomplished for the lowest notes in the higher register by half-holing the top hole of the basic six-note scale, which is one of the complications of bassoon playing. For the higher notes, other vent-holes are provided, covered with pads and opened by keys operated by the left thumb.[12]

We recall now, that p = 0 first at $x = \frac{\lambda}{2}$, next at $x = \lambda$, and next at $x = \frac{3\lambda}{2}$. This means that the third mode of oscillation of the bassoon has a frequency of three times the fundamental, i. e. a twelfth above it. It is by overblowing the twelfth that the next five pitches of the bassoon may be produced:



These pitches are not always fingered this way, because again other fingerings may be found which bring superior results in tuning and tone quality.

For the remainder of the range of the bassoon, pitch production by further overblowing is theoretically possible, but this is almost never done. Instead, individual, unrelated fingerings are used. This is not surprising when one remembers that the bassoon is very seldom called upon to produce pitches above B4.

Generally speaking, the bassoon is in every way the most irregular of the woodwind instruments. Because of its complicated physical construction, in that it is bent over on itself, there results a complicated acoustical construction, in that the lateral subdivision of the tube length is irregular. Since the tone holes for F, G#, A, c, and c#, if placed in their natural position, would occur in the socket or tenon or in the U-bend of the boot, they must be displaced. To correct the intonation it is, therefore, necessary to bore oblique holes with varying depth of penetration and diameter. Thus, additional resonance-holes are bored in the wide-bore of the butt to assist tone-holes in the narrow bore. The result all this has on bassoon playing is that overblowing succeeds in practice only as far as D4. After that, as a result of the compromises in placing the tone-holes, considerable differences in intonation and tone-quality can emerge. These differences can be compensated by auxiliary fingerings. Normally, the low tone-holes are opened or closed as far as the availability of the thumbs and little fingers permits.

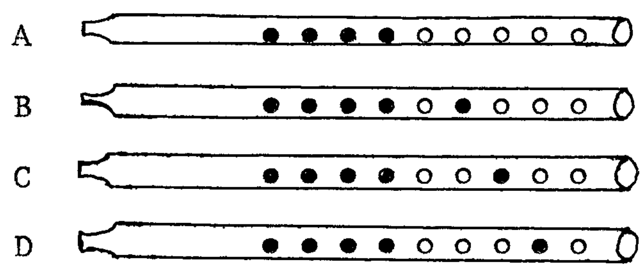
Another factor which greatly influences the acoustics of the bassoon is the taper of the bore of the crook and of the wing joint, because there the cone is the narrowest. The crook has, in fact, been called the soul of the bassoon. There is a wide variety of crooks, each having one or another advantage for the particular instrument to which it is fitted. One crook may facilitate the production of low notes, while another will suit only for producing the highest notes. Even the slightest deviations in the course of the bore result in the alteration of the theoretical length of the tube and, accordingly, upset its pitch. Theorists and even bassoonists often underestimate the influence which the construction of the tube has upon both the intonation and tone-colour of the bassoon.

Another decisive factor in determining the quality of the bassoon tone is the reed. A skilled player usually makes his own reeds or has these made by a reed-maker to suit him. Such reeds may nevertheless be unsuitable for another player, because he may have a different embouchure, different breath control, and different breath volume. The quality of a reed cane, its place of origin, and its age all play an important part. Finally, a player should have a sound knowledge of the acoustical conditions under which the reed sets in vibration the air-column in the instrument. With long experience of wind technique, a skilled player almost unconsciously produces with the reed all frequencies necessary to obtain all the notes of the 3½-octave range of the bassoon. There is really no generally suitable model reed, but one generalization may be made: the frequency of the reed itself in making it "croak" should produce a pitch somewhere between F3 and G#3.

More may be said about the tone colour of the bassoon. The lengthening of the bassoon tube produces the result that although the fundamental is always heard, it has only a fraction of the power of the entire sound volume. Measurements have been taken of the strength of the partials in the individual registers and it was found that, for example, from Bb1 to G2 the fifth harmonic has about 60% of the power, while the fundamental has only 2%. From G#2 to E3 the third harmonic has 60% of the whole power, and the fundamental only about 4%. From C#2 to C4 the second harmonic has 95% and the fundamental 3%; from C#4 to C5 the first harmonic (i. e. the fundamental) about 90% and the other harmonics the remainder of the power. The conclusion is that the prominent partials with the greatest power lie between 350 and 500 cycles per second. Tonal spectra have shown that the bassoon has a particularly strong formant precisely in this region -- at about 500 cycles per second (hertz.) [13]

It is, therefore, obvious that the tone-colour of the bassoon is actually different in the three octaves. Measurements of resonances in the bassoon show that for the low tones of the instrument, several resonances appear, lying reasonably close to the harmonic frequencies. As the scale is ascended, the resonances become fewer in number and depart more from the harmonic frequencies. Above the break in the bassoon there is again little correspondence between the resonances and the harmonic frequencies.

In the bassoon, that portion of the instrument below the first open tone hole has a considerable influence on the resonances of the portion of the instrument producing the tone, much more so than in the other woodwinds. Closing or opening a tone hole in the lower part of the bassoon can make quite a change in the quality of a tone higher up on the instrument. For this reason bassoonists have a number of fingerings available to improve the quality of certain bassoon tones.[14] The most common form of auxiliary fingering used is that of cross-fingering. Many experiments have been performed in the area of cross-fingerings. These began with most accurate measurements both as to pitch and dimensions in good existing instruments -- in the first place with the clarinet. Starting about the middle of the compass with the first four holes closed (A), it was found that closing the sixth hole (B) would appreciably flatten the note sounded with A. Closing the seventh hole (C) and reopening the sixth produced much less flattening, while closing the eighth hole (D) had almost negligible effect:



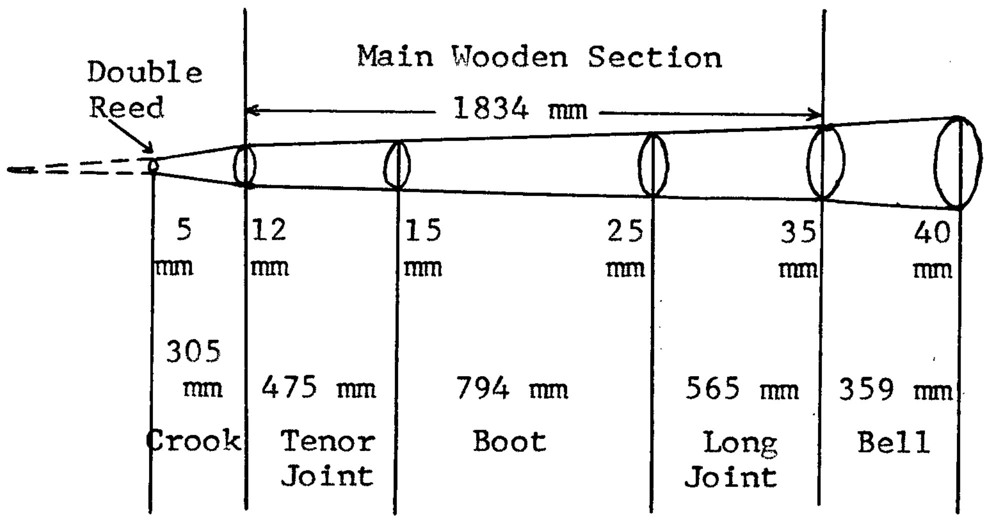
The conclusion is that the flattening due to closing a lower hole decreases the further away it is from the topmost open hole.[15] Unfortunately, tones produced in this way are not always of good quality, and often adjustments must be made through the use of other holes and keys. The reason for the poor quality of the tones produced by cross-fingering is the effect on the higher modes of the portion of the tube below the first open hole. These higher modes of the air column are pushed even further away from the harmonic frequencies than they are normally, so the harmonic content of the tone is further reduced; this results in poorer tone quality.

Much more work needs to be done on the bassoon as well as the other woodwinds to work out the relationships between the structure of the instrument, the positions of the resonances, and the harmonic structure of the tone. At present, we are not only ignorant of these relationships, but we do not even know in physical terms the difference between a "good" and a "bad" tone.[16]

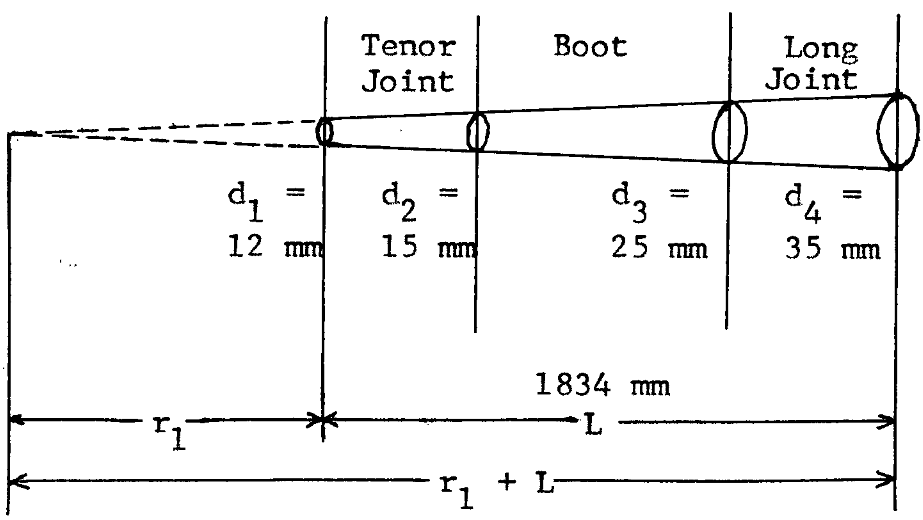
Testing the Equation $v = f\lambda$[17]

As was done with the other three instruments, we now want to test the equation $v = f\lambda$ in a practical situation involving the bassoon. We must first, however, calculate the (theoretical) length of the conical tube of the bassoon by extrapolation of the bore to its imaginary peak. Firstly, we "straighten out" the tube, and find that it looks as follows:

Now the measured dimensions (lengths and diameters) of the parts of the bassoon are as follows:



As was done with the oboe, we will first use only the most uniform, main wooden section for our extrapolation, and ignore the bell and the crook.



With $d_1 = K(r_1)$, let $d_1 = 12$ and $d_4 = K(r_1 + L) = 35$.

Then $KL = d_4 - d_1 = 23$, $\therefore K = \frac{d_4 - d_1}{L} = \frac{23}{1834} = .0125$

To verify the value obtained for K above, we can take the joints individually, as follows:

Tenor Joint:

let $d_1 = K(r_1) = 12$ and

$d_2 = K(r_1 + 475) = 15$

Then $K(475) = 3$

$\therefore K = \frac{3}{475} = .0063$

Boot:

let $d_2 = K(r_2) = 15$ and

$d_3 = K(r_2 + 794) = 25$

Then $K(794) = 10$

$\therefore K = \frac{10}{794} = .0126$

Long Joint:

let $d_3 = K(r_3) = 25$ and $d_4 = K(r_3 + 565) = 35$.

Then $K(565) = d_4 - d_3 = 10 \therefore K = \frac{10}{565} = .0177$.

Thus K for the main wooden section averages out to be

$$\frac{475(.0063) + 794(.0126) + 565(.0177)}{1834} =$$

$$\frac{2.9925 + 10.0044 + 10.0005}{1834} = 0.125,$$

i. e. identical to the value obtained above, but shown not to be uniform throughout the three joints.

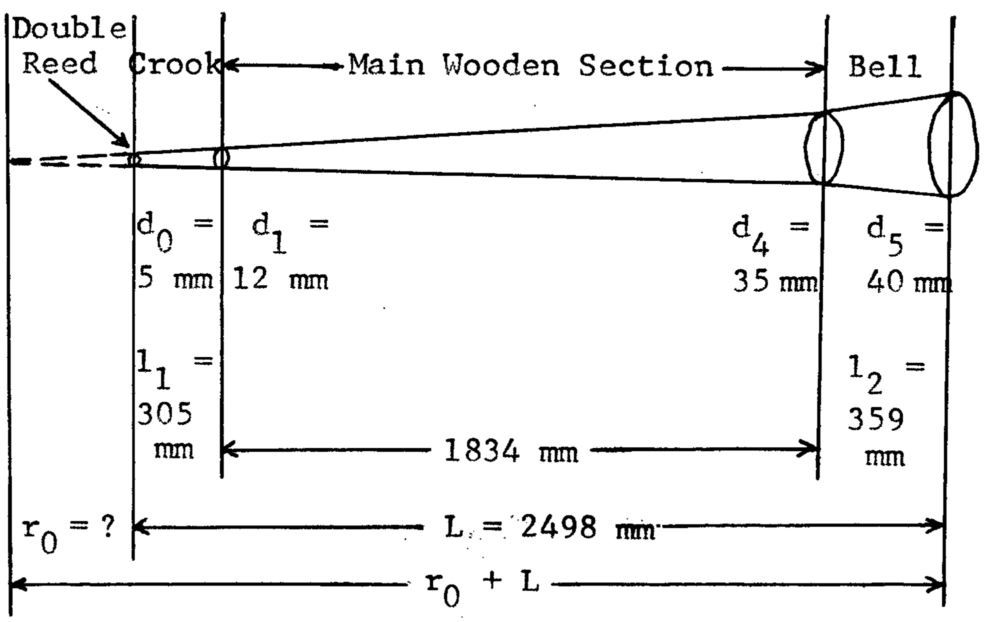
Now taking $K = .0125$, for $d_1 = K(r_1)$ we obtain:

$$12 = .0125r_1 \therefore r_1 = \frac{12}{.0125} = 960 \text{ mm},$$

and for $d_4 = K(r_1 + L)$ we get: $35 = .0125(r_1 + L)$

$$r_1 = \frac{35}{.0125} - L = 2800 - 1834 = 966 \text{ mm.}$$

Thus r_1 averages out to be approximately 963 mm (including the crook). This leads us to believe that this extrapolation is not appropriate, since the theoretical length of the tube comes out to be too great in proportion to its practical length. We must, therefore, assume that, contrary to the case with the oboe, the bell and the crook of the bassoon play a decisive role in determining the theoretical length of its cone. We now, therefore expand the previous diagram to include the entire bassoon cone, which we will use for the extrapolation:



To find the value of K for the crook, we let

$d_0 = K(r_0) = 5$ and $d_1 = K(r_0 + l_1) = 12$. Then

$$Kl_1 = d_1 - d_0 = 7, \therefore K = \frac{d_1 - d_0}{l_1} = \frac{7}{305} = .0229.$$

From above, we know that K for the Main Wooden Section averages out to be .0125. To find the value of K for the bell, we let

$d_5 = K(r_5) = 40$ and $d_4 = K(r_4 + l_2) = 35$. Then

$$Kl_2 = d_5 - d_4 = 5, \therefore K = \frac{d_5 - d_4}{l_2} = \frac{5}{359} = .0139.$$

Thus K for the entire bassoon tube averages out to:

$$\frac{305(.0229) + 1834(.0125) + 359(.0139)}{2498} =$$

$$\frac{6.985 + 22.925 + 4.9901}{2498} = .014.$$

Now, taking K = .014, for $d_0 = K(r_0)$ we get:

$5 = .014(r_0)$, $\therefore r_0 = 357$ mm, and for $d_5 = K(r_0 + L)$

we get: $40 = .014(r_0 + 2498)$, $\therefore r_0 = \frac{40}{.014} - 2498$

$$= 359 \text{ mm}.$$

$\therefore r_0$ averages out to be 358 mm, which is an appropriate proportion to the practical length of the tube. Now $r_0 + L$, the theoretical length of the bassoon cone, becomes 2854 mm.

Now we may proceed with testing the equation $v = f\lambda$ in the case of the bassoon. With a bassoon tube of theoretical length L, $L = \frac{n\lambda}{2}$ must be satisfied, therefore $\lambda_{max} = 2L$ (see page 59). For the four pitches whose corresponding wavelength was measured, the measurement was taken to the top of the tenor joint (i. e. to the point at which the diameter of the tube is 12 mm), as shown in line "a" below. When we add to each of these lengths 305 mm for the crook plus 358 mm for r_0, we get the results in line "b", which are then taken to be the length L in the table on the next page.

Pitch:	Bb1	G2	F3	C4
a) Distance from top of tenor joint to first open hole (cm):	195.9	99.4	23.8	7.3
Crook length (cm):	30.5	30.5	30.5	30.5
r_0 (cm):	35.8	35.8	35.8	35.8
b) L (cm):	262.2	165.7	90.1	73.6

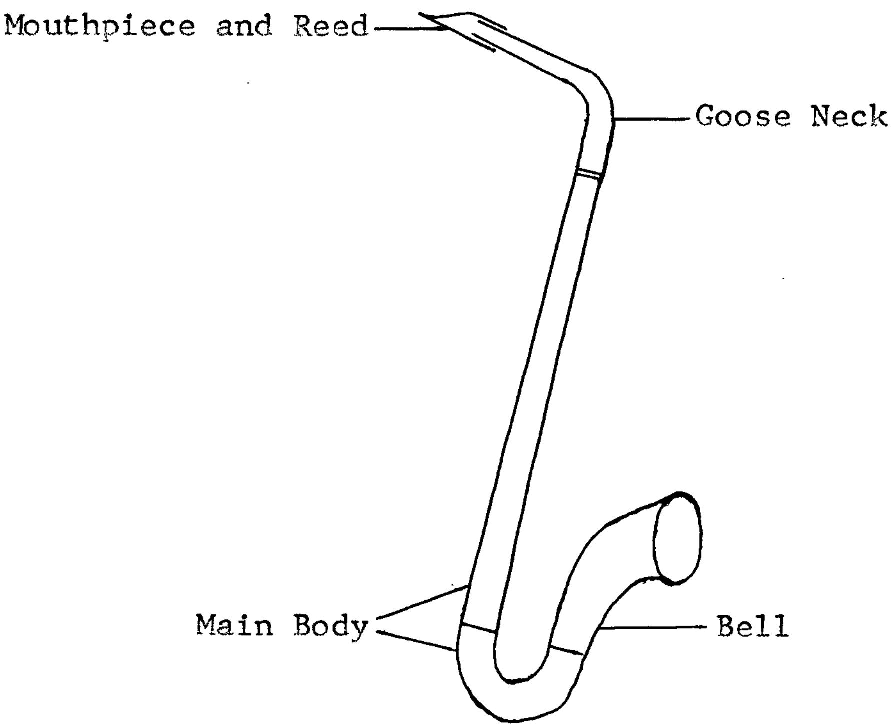
<u>Conclusion:</u> When we substitute for r and λ for the pitch C4 in $Kr = \frac{2\pi r}{\lambda}$, we get $\frac{2\pi(358)}{736}$, which equals approximately 1.53, which is not negligible in comparison to one. Since the results above come so

1a	Examined Pitch:	Bb1	G2	F3	C4
1b	Known frequency of this pitch (hz):	58.27	97.99	174.61	261.63
	Length of pipe (L) (cm):	262.2	165.7	90.1	73.6
	λ (where $\lambda_{max} = 2L$) (cm):	524.4	331.4	180.2	147.2
	Calculated frequency (33853 cm/sec $\div$ 4L) (hz):	64.56	102.15	187.86	229.98
2a	Pitch whose frequency in equal temperament is nearest to that obtained above:	C2	G#2	F#3	Bb3
	Known frequency of this pitch (hz):	65.41	103.83	185.00	233.08
	Discrepancy of 2a with respect to 1a (by interval):	+ 2nd hghr.	- 2nd hghr.	- 2nd hghr.	+ 2nd lower
	Ratio of calculated frequency with respect to 1b:	1.108	1.042	1.076	.879

close to the ones desired, we must conclude, as we did in the case of the oboe, that for small changes from the ideal conical shape of the tube, we can work with equivalent volumes whether the lengths involved are equivalent or not.

The Saxophone

The saxophone is one of the few instruments which was "invented", as opposed to being evolved. Its inventor, Adolphe Sax, who was famous for construction of both brass and woodwind instruments, wished to cross the two families and come up with a brass clarinet with a conical bore and using simple fingerings. He did this by combining the single reed of the clarinet with a brass conical bore body and with woodwind-type fingering mechanisms.[18] The basic design of the instrument has never been changed, although many improvements have been made, one of which has been a considerable widening of the original range. Theoretically, all saxophones except the soprano consist of four sections: the Goose Neck (an L-shaped tube to which the mouthpiece and reed are attached), the Main Body, the Bell, and a U-shaped tube connecting the Bell to the Main Body. Practically, though, there are only two sections, because the U-shaped tube is permanently attached to both the bell and the body, thus these three pieces form one long section. In this discussion, the U-shaped tube will be considered as a portion of the Main Body.



Holes and keys are found throughout the entire length of the instrument.

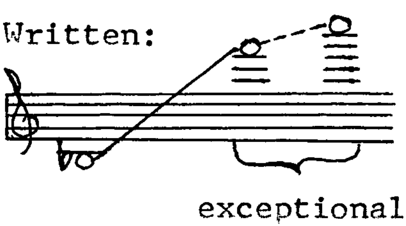
The family of saxophones in use today consists of five instruments of various sizes: Soprano in Bb, Alto in Eb, Tenor in Bb, Baritone in Eb, and Bass in Bb. All members of the saxophone family have the same fingering system -- the structural difference is mainly one of size. Because of the great similarity of these instruments to one another, only one will be considered here: the Eb Alto.

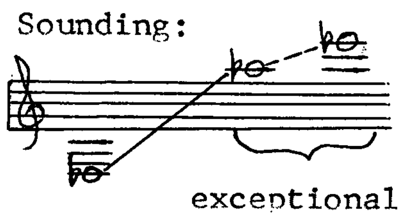
Since the conical tube closed at one end vibrates with modes that are integral multiples of the fundamental, the second mode of oscillation of the saxophone has a frequency of twice the fundamental:

$p = 0$ first at $x = \frac{\lambda}{2}$, next at $x = \lambda$.[19]

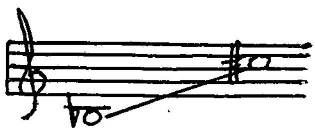
Musically, this results in a rise in pitch of an octave, which is the basic interval by which the saxophone overblows. This property, plus the fact that the saxophone has a pressure node (and not an antinode) at the mouthpiece, makes the construction and fingerings of the saxophone relatively simple. The saxophone has 12 "additional" holes and keys, which is about the same number as the oboe.

The basic scale of the Eb Alto saxophone is the six-hole, seven-note scale, which starts on D4 with the six holes closed. This D4 sounds as F3, a major sixth lower than written. Additional holes and keys on the bell are used to extend the range down to (written) Bb3. The normal range of the saxophone, then, is the following:





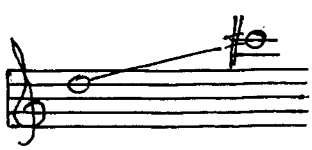
The written range shown here is that of all saxophones; the sounding range shown is that of the Eb Alto saxophone. The pitches from Bb3 to C#5 have their own individual fingerings:[20]



The basic scale proceeds: D4, E4, F4, G4, A4, B4, C#5, and the chromatic notes in between are produced through the use of additional holes and keys.

There are two ways of producing C#5. The first, which is conventionally used, is with all holes open, i. e. as the last note of the basic scale. The second (and worse) method is by overblowing C#4 at the octave. The pitches Bb4, B4, and C5 may also be produced by overblowing at the octave, but the tone quality is inferior, so this is not done.

The first break of the saxophone occurs between C#5 and D5. Beginning with D5, octave overblowing is used for all pitches up to C#6:

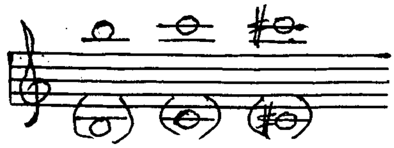


In each case, the fingering is identical to that used to produce the pitch an octave lower, except that the octave key is added.

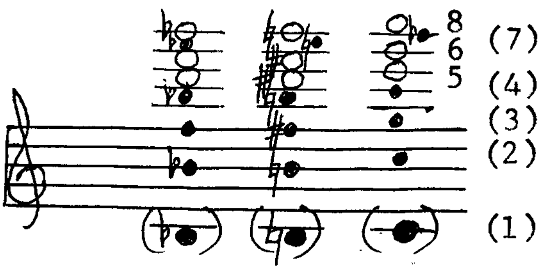
We recall now, that $p = 0$ first at $x = \frac{\lambda}{2}$, next at $x = \lambda$, and next at $x = \frac{3\lambda}{2}$. This means that the third mode of oscillation of the saxophone has a frequency of three times the fundamental, i. e. a twelfth above it. In the case of the saxophone, however, there is only a limited range in which overblowing by the twelfth is possible. From F5 to A5 this can be done quite readily, but from Bb5 to C6 it is more difficult:

Furthermore, since all these pitches are best produced through the second mode of oscillation, as shown above, overblowing by the twelfth is a resource which is seldom used on the saxophone. The last three pitches of this register, however, may be produced by use of the fourth mode of oscillation, (where

$x = 2\lambda$), i. e. by overblowing at the double octave:



Between C#6 and D6 is where the next break of the saxophone occurs. From here on up, the best method of pitch production from the <u>acoustical</u> standpoint is derived from the overtone series on Bb3, B3, and C4, the three lowest pitches of the instrument:

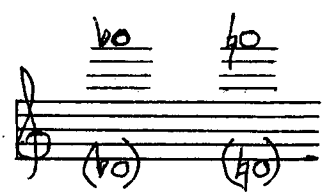


When we continue the previously given series of values for which $p = 0$, we get the following:

$p = 0$ when $x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda, \frac{5\lambda}{2}, 3\lambda, \frac{7\lambda}{2}, 4\lambda,$ etc... When $x = \frac{5\lambda}{2}$, the instrument is operating in its fifth mode of oscillation, i. e. with a frequency of five times the fundamental, or two octaves plus a major third above it. This is how the first three pitches of this register, D6, D#6, and E6, may be produced.

This pattern is repeated for the sixth mode of oscillation, i. e. for $x = 3\lambda$, where the resulting pitch is six times the fundamental, or two octaves and a perfect fifth above it. This is the case for the next three pitches, F6, F#6, and G6.

When we come to the seventh mode of oscillation (where $x = \frac{7\lambda}{2}$), we encounter an inevitable problem: the resulting pitches turn out flatter than the desired ones because of the nature of the harmonic series. Rather than sounding two octaves plus a minor seventh above the fundamental, the resulting pitch frequently sounds at an interval of two octaves plus a sharp major sixth. Consequently, we are forced to seek another method of producing Ab6, A6, and Bb6. For Ab6 and A6, the best solution is to return to the sixth mode of oscillation and overblow Db4 and D4 by two octaves and a perfect fifth:

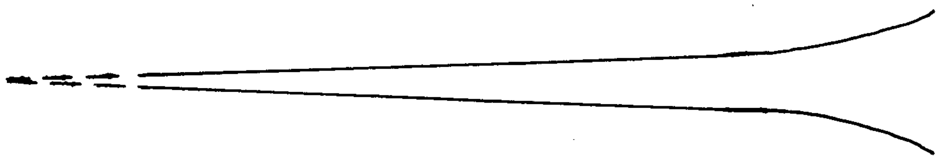


For Bb6, our problem has a much simpler solution. We simply group it with the next two pitches, B6 and C7, and produce all three of these pitches through the eighth mode of oscillation, where $x = 4\lambda$, i. e. by overblowing at the triple octave, as shown in the previous example.

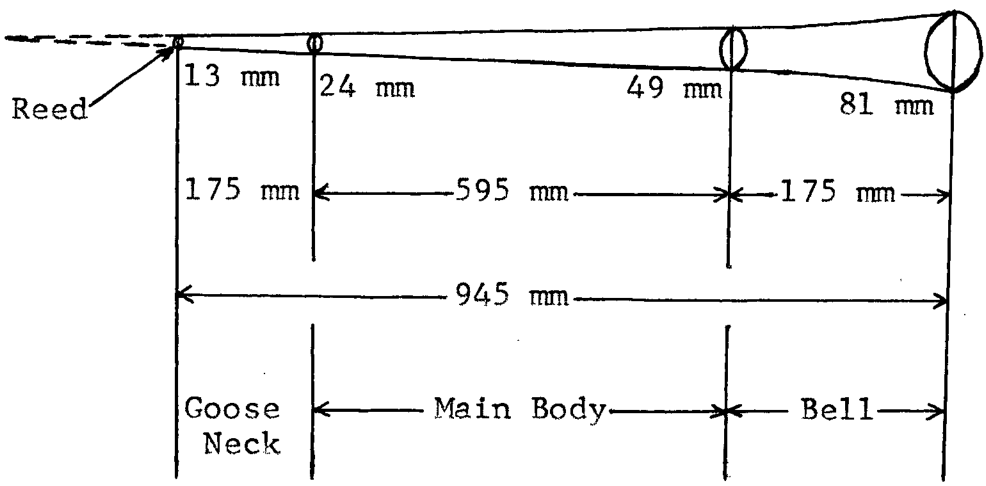
It must be stressed here, that this method of pitch production is valid from the acoustical standpoint, but from the practical standpoint, saxophonists often find their own "artificial" fingerings, which either suit them better, or suit their instrument better, or simply give more satisfactory results. One should not be surprised, therefore, to find many different usable fingerings for the pitches in the highest register of the saxophone.

Testing the Equation $v = f\lambda$ [21]

We now want to test the equation $v = f\lambda$ in a practical situation involving the saxophone. With a saxophone of length L, $L = \frac{n\lambda}{2}$ must be satisfied (see page 59). We must first, however, calculate the theoretical length of the conical tube of the saxophone by extrapolating the conical bore to the imaginary peak. Firstly, we "straighten out" the tube, and find that it looks as follows:

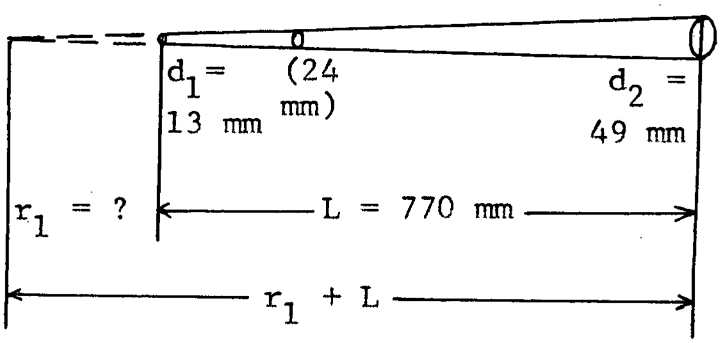


Now the measured dimensions (diameters and lengths) of the saxophone are as follows:



Since the bell is permanently attached to the main body, the diameter of the bore at that point had to be estimated as follows: The external circumference was measured and found to be 173 mm. 173 $\div \pi$ gave 55 mm. The thickness of the tube wall was estimated at 3 mm, therefore 6 mm were subtracted for a final estimate of 49 mm.

Since the bell of the saxophone flares out much more rapidly than the remainder of the bore, we shall not consider it in the extrapolation, and use only the greater part of the tube:



With $d_1 = K(r_1)$, let $d_1 = 13$ and $d_2 = K(r_1 + L) = 49$.

Then $KL = d_2 - d_1 = 36$, $K = \frac{d_2 - d_1}{L} = \frac{36}{770} = .0468$.

Now $r_1 + L = \frac{d_2}{K} = \frac{49}{.0468} = 1047$, $\therefore$ $r_1 = 277$ mm.

To verify the value obtained for K, we can take the two sections separately, as follows:

Goose Neck:

let $d_1 = K(r_1) = 13$ and

$d_2 = K(r_1 + 175) = 24$

Then $K(175) = 11$

$\therefore K = \frac{11}{175} = .0629$

Body:

let $d_1 = K(r_1) = 24$ &

$d_2 = K(r_1 + 595) = 49$

Then $K(595) = 25$

$\therefore K = \frac{25}{595} = .0420$

Thus K averages out to be $\frac{175(.0629) + 595(.0420)}{770}$

$= \frac{11.0075 + 24.990}{770} = \frac{35.9975}{770} = .0468$, i. e. identical to the value obtained above, but shown not to be uniform throughout the tube.

For the four pitches whose corresponding wavelength was measured, the measurement was taken to the top of the Main Body (i. e. to the point at which the diameter of the tube is 24 mm) as shown in line "a" below. When we add to each of these lengths 175 mm for the Goose Neck plus 277 mm for r_1, we get the results in line "b", which are then taken to be the length L in the table on the next page.

Pitch:	Written: / Sounding:	Bb3 / Db3	F4 / Ab3	A4 / C4	C#5 / E4
(a)	Distance from top of Main Body to first open hole (cm):	77.0	39.7	23.3	10.8
	Length of Goose Neck (cm):	17.5	17.5	17.5	17.5
	r_1 (cm):	27.7	27.7	27.7	27.7
(b)	L (cm):	122.2	84.9	68.5	56.0

<u>Conclusion:</u> When we substitute for r and λ for the

1a	Examined Pitch: Written: Sounding:	Bb3 Db3	F4 Ab3	A4 C4	C#5 E4
1b	Known frequency of this pitch (hz):	138.59	207.65	261.63	329.63
	Length of pipe (L) (cm):	122.2	84.9	68.5	56.0
	λ (where $\lambda_{max} = 2L$) (cm):	244.4	169.8	137.0	112.0
	Calculated frequency (33853 cm/sec $\div$ 2L) (hz):	138.51	199.37	247.10	302.26
2a	Pitch whose frequency in equal temperament is nearest to that obtained above:	Db3	G3	B3	D4
	Known frequency of this pitch (hz):	138.59	196.00	246.94	293.66
	Discrepancy of 2a with respect to 1a (by interval):	none	- 2nd lower	- 2nd lower	+ 2nd lower
	Ratio of calculated frequency with respect to 1b:	.999	.960	.944	.917

pitch C#5 in $Kr = \frac{2\pi r}{\lambda}$, we get $\frac{2\pi(277)}{1120}$ which = 1.55, which is not negligible in comparison to one. Since the results come so close to the ones desired, we must conclude that for small changes from the ideal conical shape of the tube, we can work with equivalent <u>volumes</u> whether the lengths involved are equivalent or not.

One further observation may be made from the results obtained. Since the discrepancy between the calculated frequency and the known frequency is systematic rather than uniform, this means that r_1 has an increasingly greater effect on this discrepancy for increasingly higher pitches. This implies that it would be possible to make a better choice of the value of r_1, so that the discrepancy would, instead, be constant for the entire range of the instrument. Thus, although it would be slightly greater for the lowest pitches, it would be far smaller for all the higher pitches than it is presently.

IV. THE BRASS INSTRUMENTS [1]

Like the woodwinds, the brass instruments have their origin in prehistoric times. Originally, these instruments were nothing more than seashells or hollow animal horns, later, with the development of techniques and skills of metal working, these shapes were reproduced in copper or brass. As time went on, alterations and improvements were made according to musical requirements, until the instruments acquired the shapes which they have today.

Acoustically, the brass instruments might be classified with the woodwinds as brass wind instruments, however, they differ in enough important respects from the woodwinds to be considered a separate family. Surprisingly, the least important difference is that which is inherent in the names of the two groups of instruments, which emphasize the material of construction -- metal as opposed to wood. We must remember, that one of the modern woodwinds, the flute, is not wooden at all, but metal, and another woodwind, the clarinet, may also be constructed of materials other than wood. The fundamental differences, then, are the following:

First, the vibrations of the air column in the brass instruments are maintained by the vibrations of the player's lips, instead of by air streams or reeds. Since the mass of the lips is considerably greater than that of woodwind reeds, the lips can influence the air column vibrations more easily.

Second, the brass instruments use many more resonance modes of the air column than the woodwinds do. Some brass instruments use only resonance modes; the majority, however, have a complete scale, which is made available by filling in the gaps between modes.

Third, the principle on which brass instruments are constructed is the exact opposite of the side-hole system of the woodwinds. Instead of effectively shortening the air column, it is progressively lengthened by the addition of extra lengths of tubing. This is achieved by means of either a movable telescopic section in the main tube, or by a number of spring-loaded valves, each of which, when depressed, switches in a preadjusted length of supplementary tubing and cuts it out again when released. The telescopic system is found today only in the slide trombone, but it is the older and simpler of the two systems.

Fourth, since there are no open holes in the sides of brass instruments, all the sound must come out of the bell. Consequently, the size and shape of the bell play a decisive role in the acoustics of brass instruments, whereas in the woodwinds the influence of the bell is not that great.

Fifth, there is a fundamental difference in the acoustical foundations of the two groups of instruments. We know now that the woodwind instruments utilize the well-defined, harmonically related resonance frequencies of cylindrical and conical tubes. The brass instruments, on the other hand, cannot be divided so distinctly into two such groups. Instead, they are all partly cylindrical and partly conical, which brings us back to the point mentioned above, stressing the acoustical importance of the construction of the bell.

The sixth and final difference between the two families of instruments lies in the significance of their deviations from basic cylindrical and conical shapes. In woodwind instruments, the presence of covered tone holes and other factors results in unavoidable deviations of the bores from perfect cylindrical or conical shapes. These deviations shift the resonance frequencies away from the harmonic frequencies, which leads to intonation problems and the necessity of getting around these by artificial means. In short, unavoidable deviations from the original shapes of the tube are a basic source of problems in the construction and fingerings of the woodwind instruments. On the other hand, in the brass instruments, the opposite is true. Distortions from the simple shapes are introduced deliberately and in a large degree in order to produce a musically useful series of resonance modes. We will now go on to consider how this is done.

If the lips are placed against a smooth ring and put under some tension, they can be made to vibrate by blowing air through them from the lungs, thus producing a buzzing sound. Since the mass of the lips is relatively large, the vibration frequency produced is relatively low. If the tension of the lips is increased, they will vibrate at a higher frequency, just as vibrating strings do.

Now if we attach a (cylindrical) tube to this ring, the vibrating lips will build up oscillations of that particular resonance mode of the air column which has a frequency near that of the lips. The mechanism for producing vibrations in this system is then basically the same as that for the reed instruments, except that here the lips play the role of the reed. The important acoustical consequence of this similarity is that there is a pressure antinode in the air column at the lip end, where energy is being supplied.

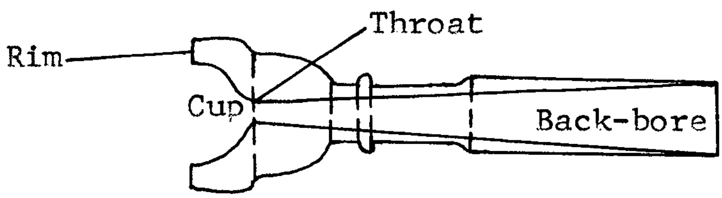
The frequency of the combination of lip and air column will be primarily determined by the frequency of the particular mode excited. However, because of the relatively large mass of the lips, which are here playing the role of the reed, the influence of the lips on the vibration frequency is much more considerable than it is in the woodwinds. For this reason, increasing the lip tension can excite the higher modes of vibration of the air column fairly easily; thus the vent-holes needed on the woodwinds are not necessary on brass instruments. This makes it possible to use many more of the resonance modes of the air column than may be used in woodwinds.

Since there is a pressure antinode at the lip end of the column, this end behaves like a closed end. Consequently, the general equation involving λ for such a tube of length L is $L = \frac{\lambda}{4}$, which means that $p = 0$ when $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}$, etc... This tube then vibrates with modes that are odd multiples of the fundamental, as does the clarinet. The conclusion is that such a tube does not have a musically useful series of modes.

From the musical standpoint, the tube has another fault which so far has not been mentioned. The tone it produces when blown with the lips is muffled and of poor quality -- nothing similar to a good brass tone. For these two reasons, the simple cylindrical tube needs much alteration and improvement to make it into a useful musical instrument. Two basic alterations have been made which succeed in achieving this goal: a mouthpiece has been added to the lip end of the tube to replace the ring, and the opposite end of the cylindrical tube has been flared into a bell. The precise shape of both is of crucial importance.

The Mouthpiece [2]

The mouthpiece consists of a small cup, with a rim designed to fit against the lips. The cup is joined at the throat to a tapering tube of considerably smaller diameter than the rest of the instrument; this small tube is called the back-bore. The following diagram shows a section through a mouthpiece:



The width of the rim is a matter of comfort, but an excessively wide rim will reduce flexibility and may even disturb tone quality because of the unnatural position which it enforces upon the embouchure. A large mouthpiece tends to produce a bigger tone, but not every player's embouchure is strong enough to use such a mouthpiece. The depth of the cup is of great importance; the deeper the cup, the darker and more solid the tone. However, a deep cup also makes the high register more difficult to play in. Also, the shape of the cup, whether it is a real cup or even a funnel, will affect quality and ease of playing. The bore is of great importance; a large bore offers a minimum of resistance and a maximum of tone, but not all players have the physical stamina required by such a mouthpiece; furthermore, such a mouthpiece is difficult to work with in extreme piano passages. The back-bore also has a bearing on quality and playing characteristics; a cone with a high K factor in the back-bore tends to facilitate playing, but may cause the instrument to become rough in the forte passages, while a more cylindrical back-bore offers greater resistance, resulting in a more solid tone which will not speak quite so readily.

If the mouthpiece shown above is attached to the tube we have been considering and the cup is closed off by the lips, the tube will effectively be lengthened. At low frequencies, where the values of λ are large in comparison to the size of the mouthpiece, it turns out that the amount of lengthening is exactly that of a piece of the tube having the same volume as the mouthpiece. The higher resonances, however, do not remain unchanged when the mouthpiece is added. The combination of cup + tube has a certain resonance frequency, and at this frequency it behaves the same as a tube closed at one end that has this as its fundamental frequency. The result is that the mouthpiece lengthens the tube by an amount that increases as the frequency increases. Consequently, the tube with the mouthpiece attached has the same lowest resonance frequency as before, but the high modes are shifted downward from their original values, a desirable result.[3]

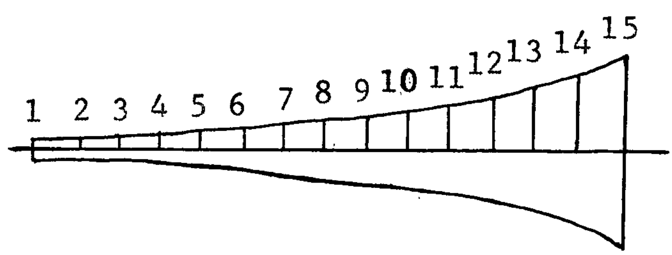
In general, the mouthpiece is probably the greatest single factor in tone quality and proper tone production in brass instruments. A below-average instrument can be made to sound fairly good with a good mouthpiece, and an excellent instrument can be made to sound inferior with a bad one.

The Bell

Just as we wanted to shift the high modes of the plain cylindrical tube down, our second desirable improvement is that its low modes move up. This is accomplished by flaring the open end of the tube by increasing its diameter as the open end is approached, thus forming the familiar bell of brass instruments. At the same time, in order to leave the higher resonances unchanged, the total length of the instrument must be properly adjusted. The actual amounts by which the lower modes will come up depends on the detailed shape of the bell. With a properly shaped bell (and mouthpiece), the resonance frequencies will be shifted into those modes we conventionally associate with a brass instrument.

Adding the bell to a brass instrument has another very important effect: the tone it produces is much louder and clearer than that produced by the cylindrical tube without the bell; this is our third desirable improvement. There are two reasons for this effect. First, as might be expected, the amount of sound which radiates from the open end of a vibrating air column depends on the area of the open end. Thus the bell of a brass instrument serves the very important function of increasing the sound output of the instrument. Second, the production of harmonics in the tone is facilitated. When we sound a note in the simple tube, none of its harmonics lies very close to a resonance, therefore their production is poor. For a trumpet, on the other hand, the harmonics of a note all coincide very well and give the tone its characteristic brass timbre.[4]

Let us now take a closer look at the actual construction of the bell. All modern brass instruments are provided with bells ending in a short section or "skirt", which expands much more rapidly than the tapered portions of the main tube. The main taper of the majority of brass instruments develops in what is called "exponential" form, in which the ratio of successive radii at unit distances apart along the tube is a constant. The following figure shows the section along an "exponential horn" in which this constant quantity is 17/14:



This ratio has been called the "flare coefficient", which can be regarded as a measure of the expansion of such a tube.

The following table shows approximate flare coefficients for four modern brass instruments. It is remarkable that three of them are identical.

Instrument	Flare Coefficient
French Horn	1:25
Cornet	1:25
Trumpet	1:25
Trombone	1:3

This seems to be hardly accidental and implies that the coefficient of 1:25 has been arrived at by manufacturers through practical experience. Although at first sight the french horn appears to have a very wide bell, the exponential flare is, in fact, continuous from one end of the instrument to the other.[5] The flare coefficient of the trombone varies so drastically because the trombone is the only instrument employing the sliding-valve principle, which demands that it be perfectly cylindrical for the greater part of its length. To compensate for this, the part that flares out does so at a high ratio.

Having considered the bell, let us now look briefly at muting. It is a commonly held view that the sole purpose of a mute is to reduce the volume of sound eminating from an instrument. This, however, is not the whole story, because although reduction in volume does occur, the real function of the mute is to modify a complex tone by disturbing the normal distribution of its partials. By muting, some partials may be reduced in relative intensity while others (often the more dissonant ones) are made more prominent. The mute does this by altering the physical character of the bell, which governs the sound to a large extent. The insertion of the player's hand into the bell has a similar effect.[66] Unlike primitive mutes, modern mutes do not, as a rule, raise the pitch.

A short discussion of timbre is also appropriate at this point. The factors which influence the timbre of brass instruments have been recognized for a long time. There are basically five, the first two of which have already been mentioned:

1) The construction of the mouthpiece
2) The curvature and size of the bell
3) The ratio of the diameter of the tube to its

length, i. e. the scale of the tube

4) The shape of the tube (cylindrical, conical, or mixed)

5) The thickness of the tube walls.[7]

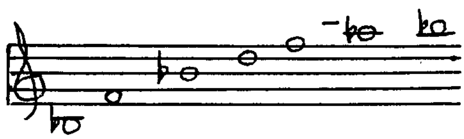
The Trumpet

The trumpet consists essentially of three parallel lengths of straight brass tubing connected with two bends, which together form one long oval coiled one and a half times around. The three sections may conveniently be called the leadpipe (to the end of which the mouthpiece is attached), the middlepipe, and the bellpipe. The first two of these sections are cylindrical and the last conical. The flare of the bellpipe actually begins somewhere in the second bend.

Since the bore of the trumpet, like that of all brass instruments, is neither entirely cylindrical nor entirely conical, but a combination of the two, the trumpet does not behave like an instrument belonging to either the cylindrical or the conical group. It does, however, lend itself easily to overblowing and to operation in at least 8 -- and in the hands of good players up to 16 -- modes of oscillation. More importantly, its successive modes of oscillation coincide with the harmonic series beginning with the second harmonic. For practical purposes, it is best to take the eighth harmonic as the last accessible one. Consequently, the trumpet, as described thus far, is naturally capable of producing the following pitches:

Written:

Sounding:



A word of explanation is in order here regarding the absence of the first harmonic. As was explained in the previous section, when the end of a cylindrical tube is flared into a bell, the low modes of the tube are moved up. In the case of the trumpet, the second, third, and fourth modes end up in the proper places, corresponding to the respective members of a harmonic series. The fundamental, however, is not shifted up far enough, and ends up being more than a perfect fourth below the position of the first harmonic of the series. For this reason, the first mode of oscillation of the trumpet is not used. [8]

In order to provide the trumpet with a complete scale, the intervals between the notes shown above must be filled in. The largest interval which needs to be filled in is the first one, that of a perfect fifth, which requires six additional semitones to fill the space. These are provided by lengthening the air column through the use of valves. Six additional lengths are sufficient; in addition to filling in the space between the first two modes, they allow the trumpet to play six semitones below the lowest pitch shown above, and are more than adequate to fill in the spaces between the higher modes. [9]

Three valves are used in the B-flat trumpet. The middle valve adds tubing which lowers the original pitch by one semitone. The valve closest to the player (valve 1) lowers it by two semitones, and the farthest valve (valve 3) lowers it by three semitones. Consequently, when valves are combined, the resulting shift of frequency downward (in semitones) is as follows: [10]

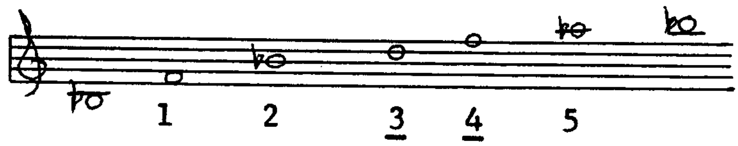
Valves	Number of semitones
1 + 2	3
2 + 3	4
1 + 3	5
1 + 2 + 3	6

Through the use of these valves, the following pitches may be produced on the trumpet as indicated: [11]

Valves **Possible (Sounding) Frequencies**

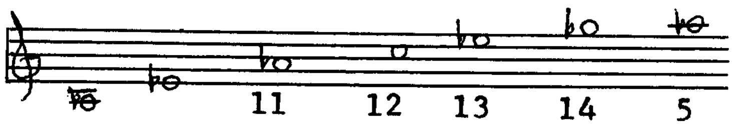
Mode: 2 3 4 5 6 7 8

0

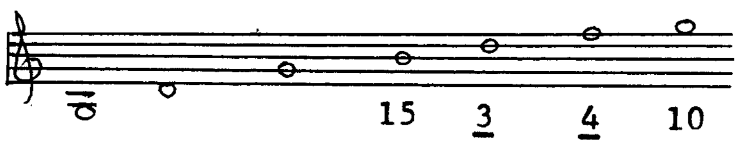


2

1

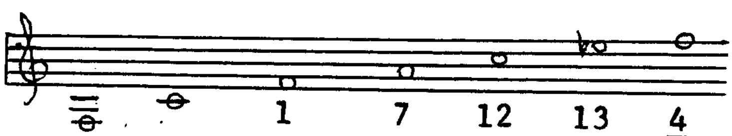


1 + 2
(or 3)

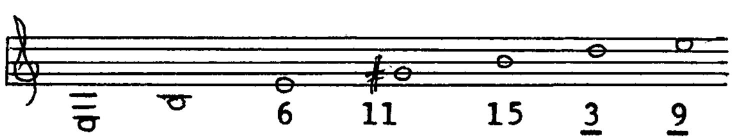


2 + 3

1 + 3



1 + 2 + 3



This is so because each fundamental which results from a new combination of valves has its own overtone series, the members of which are accessible in each case by overblowing. In general, for the fourth series shown, valves 1 + 2 are used as opposed to valve 3, for reasons to be explained later.

Although there are seven rows containing seven notes each, only 31 of these 49 notes are different. These 31 notes have been subdivided into three categories as follows:

1) 16 pitches which appear only once -- unnumbered,

2) 12 pitches which appear twice each -- indicated with the same number in both places,

3) 3 pitches which appear three times each -- indicated with the same number underlined in all three places.

The next step is to determine which valves the trumpeter will, in practice, use to produce the 31 pitches of the range of his instrument. Clearly, for the 16 pitches for which only one fingering is possible he has no choice. These pitches are indicated in whole notes in the table on page 129. For the remainder of the pitches, the process of elimination is used to determine which fingering will give the best result. Firstly, the possibilities presented in the sixth column can be safely eliminated, because these are seventh harmonics, which are considerably flatter than the corresponding note in the equal-tempered scale, and for this reason would be the worst choices to make. Elimination of the sixth column yields two results:

1) It determines the only possible fingering remaining for pitches 5, 10, 13, and 14. These pitches are indicated in half notes in the table on page 129.

2) It reduces the number of possible fingerings for pitches 3, 4, and 9 from three to two. Now there are two choices of fingerings for each of the eleven remaining pitches. For each of these, we choose the one which comes first, (reading from top to bottom); this way no more than one valve will have to be used to produce any one of these pitches. This is, in general, better than the use of two or more valves, for reasons to be given later. These last eleven pitches are indicated in quarter notes in the following table.

Valves — Sounding Frequencies Produced

Mode: 2 3 4 5 6 7 8

0

2

1

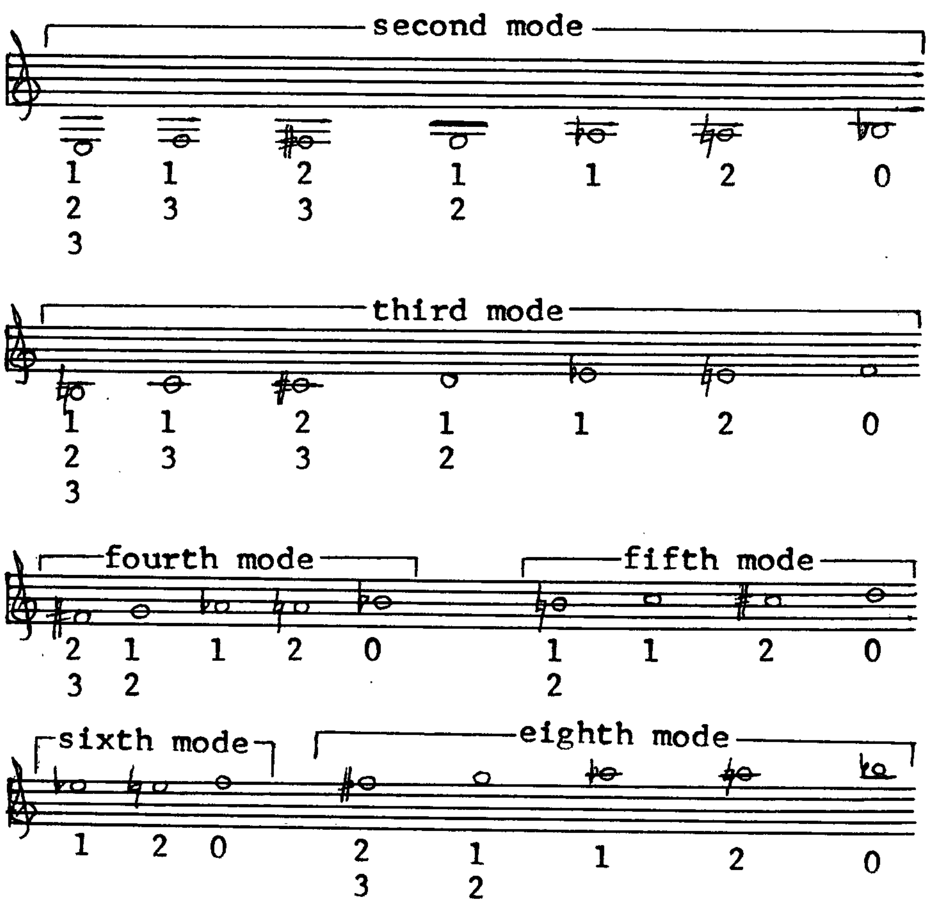
1 + 2
(or 3)

2 + 3

1 + 3

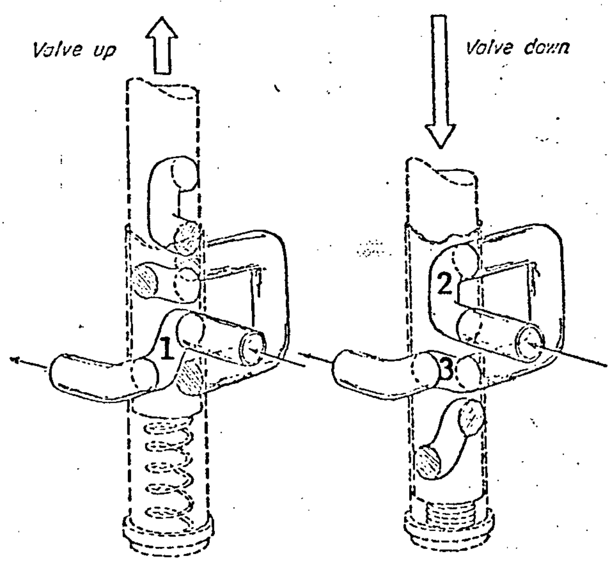
1 + 2 + 3

This table clearly shows that the use of more than one valve is necessary for at least 8, but at most 13 of the 31 pitches of the normal range of the trumpet. The complete chromatic scale of the trumpet may now be written out, showing the fingerings (by valve numbers) and the modes of oscillation used. As was explained earlier, the first mode of oscillation is not used, so we begin with the second:[12]



The Valves

Generally, two types of valve systems are in use today: rotary valves and piston valves. Since piston valves are by far the more common ones on this continent, let us examine in detail just how they operate. The mechanism of a piston valve is shown in the following figures: [13]



A cylindrical piece of metal called the piston is arranged to move up and down inside a piece of tubing permanently attached to the cylindrical portion of the main tube. The piston is normally held in the "up" position by a spring, but can be depressed with the finger. In the "up" position, a hole in the piston called the valve port (indicated "1" in the figures) carries the air column, as shown in the first figure above. In the "down" position, two other valve ports in the piston (indicated "2" and "3" in the figures) divert the air column, as shown in the second figure above, so that it passes through an additional piece of tubing. Pushing the valve down thus lengthens the air column; the amount of increased length depends on the length of the added valve tubing. This is made adjustable for tuning purposes by providing it with a movable U-shaped portion called the valve-slide.

As we have seen, three valves are used in the B-flat trumpet. While these valves work well when used individually, they are not magic, and when they are combined, a fundamental difficulty arises. If the first and second valves were designed to insert tubing of the correct length to lower the pitch by a tone and a semitone respectively, then the two valves together would not add enough tubing to lower the pitch by three semitones. Further, if all three valves were used together, they would not add enough tubing to lower the pitch by six semitones.

This problem is clearly illustrated by working out some actual numbers. Suppose that the original length of the trumpet is 150 cm. To lower the pitch produced by this tube a semitone in the equal-tempered scale, requires that the frequency be divided by a factor of $\sqrt[12]{2}$, or approximately 1.05946. This can be done by multiplying the equivalent length of the air column by the same factor, which amounts to an increase in length of 5.946%, which may be rounded off to 6%. Thus, to lower the pitch of the tube by a semitone, valve 2 would have to add 9 cm of length to the tube, giving it a total length of 159 cm. To lower the pitch by another semitone would require adding 6% of this, or 9.54 cm. This means that valve 1 would have to add 9 + 9.54 = 18.54 cm to go down two semitones. Now the effective length of the tube would be 168.54 cm. The next step down would require an addition of 6% of this length, or 10.1124 cm, which may be rounded off to 10.11 cm. But the first valve adds only 9 cm, so the combination of valves 1 and 2 is 1.11 cm short of the amount necessary to lower the pitch by a minor third. Now if we wanted to sound the pitch lying a minor third below this one, we would have to add 6% of

178.65 cm, the present length, three times over, that is $([(178.65)(106\%)][106\%])(106\%)$, which, rounded off, equals 212.77 cm. But the basic length + valve 2 + valve 1 + valve 3 give a total of 150 + 9 + 18.54 + 28.65 = 206.19 cm, therefore 6.58 cm less than needed. [14]

Such a system is, in fact, unsatisfactory, and requires modification in order to make it into a usable arrangement. It is possible to use a system in which extra pieces of tubing are automatically added to the regular valve tubing whenever the third valve is used in combination with either of the other two. In such a system, only the two pitches produced with all three valves together are slightly out of tune. Unfortunately, all the extra pieces of tubing which are required by this arrangement add considerably to the complication and expense of manufacture. Consequently, this system is not commonly employed.

Fortunately, there is another way of reducing the errors inherent in the valve system. If the third valve tube is made slightly longer than its correct acoustical length, then when this valve is used in combination with either of the first two, the total length of the tubing is much nearer to its correct value. Such a compromise closely resembles the compromise of equal temperament itself, in that the errors of intonation are reduced by being spread more widely.

With the third valve slide modified in this way, all pitches produced with the open tube, or with the first and second valve alone, will be correctly in tune. The first and second valves together will give notes approximately one tenth of a semitone sharp; the third valve used alone will give these same notes approximately one fifth of a semitone sharp. This is why the use of the first and second valves together is better than the use of the third valve alone. On the other hand, pitches produced with the second and third valves together are now very nearly in tune, and the combination 1 + 3 gives notes capable of correction by the player. The combination of all three valves together gives pitches which are about a third of a semitone sharp.[15]

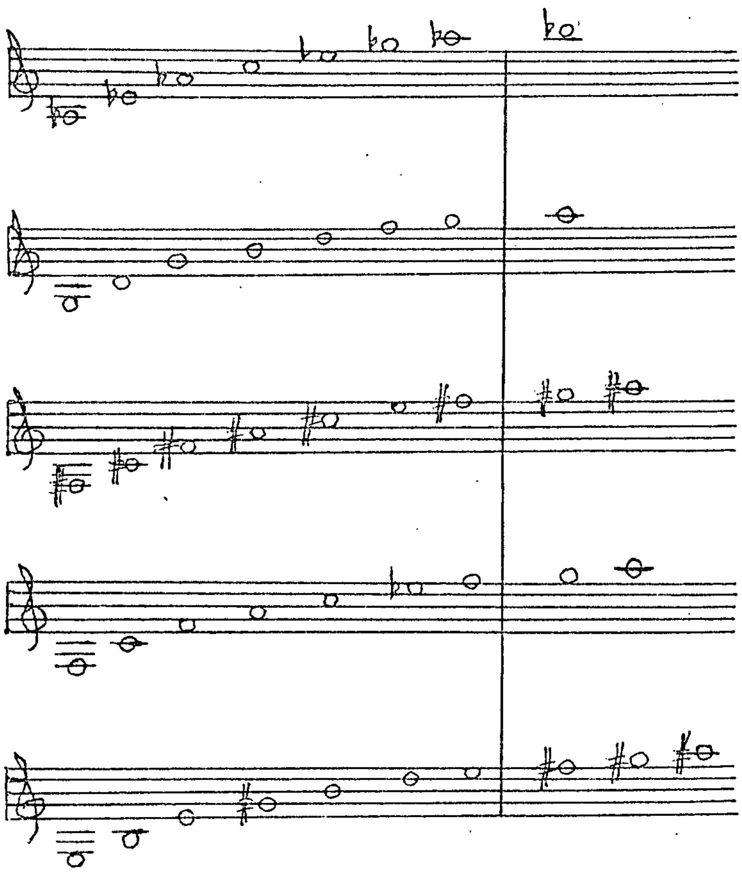
It is now clear why, when choosing fingerings, valve combinations were avoided in all cases except those involving the combination 1 + 2. Although pitches involving this combination are now well in tune, we still require improvement in intonation of pitches involving valve 3. This is especially noticeable when all three valves are used together. For this reason, on the modern trumpet, the valve slide of the third valve may be moved with the little finger of the left hand while the instrument is being played. By lengthening the slide when all three valves are down, the discrepancy may be removed.

Finally, the player can do some correcting of discrepancies with his lips, sometimes as much as 3/4 of a semitone. Lip control is also necessary to compensate for intonation problems caused by differences in temperature. If, however, lip correction is too great, the tone suffers, so it is most advisable for the player to tune his instrument to the best of his ability.[16]

Having determined the best fingerings for all the notes of the normal range of the trumpet, we can now explore other possibilities, which so far have not been considered. Taking Bb5 to be the top limit of the trumpet range, it is possible to continue the harmonic series built on E3, F3, F#3, G3 and Ab3 farther than at first, and still remain within the range:

Valves	Normally Accessible Sounding Frequencies	Additional Accessible Frequencies With Bb5 Taken as Top Limit

Mode: 2 3 4 5 6 7 8 9 10 11



1

3

2 + 3

1 + 3

1 + 2 + 3

Thus it is possible to gain three more fingerings for Bb5, two more fingerings for A5 and

G#5, and one more fingering for each of G5 and F#5. Of these, only one must be rejected, the A#5 which is the eleventh harmonic of E2, because eleventh harmonics, like sevenths, are considerably flatter than the corresponding note in the equal-tempered scale. But this still leaves eight additional fingerings, which may at times be useful in simplifying a difficult passage, as may all the alternate fingerings given earlier.

Testing the Equation $v = f\lambda$

The testing of the equation $v = f\lambda$ in practical situations involving the brass instruments will have to be carried out by a method quite different from that employed with the woodwind instruments.[17] The reason for this is simply that we do not have an equation relating λ_{max} to L for a pipe which is partly cylindrical and partly conical. Neither $\lambda_{max} = 2L$ nor $\lambda_{max} = 4L$ holds true. What we shall do, therefore, is substitute known values of v and f into the equation, which will give us λ, which we will then compare to L.

To begin with, we need to know the lengths of all the parts of the B-flat trumpet. These lengths were measured and found to be as follows:

Part	Length (in cm)
Mouthpiece	6.3
Leadpipe	33.0
First Bend	16.5
Middlepipe	20.7
Bell Section	63.5
Basic Length (equals total of above lengths)	140.0
Tubing added by valve 1	20.3
Tubing added by valve 2	12.2
Tubing added by valve 3	31.7
Tubing added by valves 1 + 2	32.5
Tubing added by valves 2 + 3	43.9
Tubing added by valves 1 + 3	52.0
Tubing added by valves 1 + 2 + 3	64.2

From these values we may easily find, by referring back to the table on page 129 and by addition, the total length of the pipe (L) used to produce the seven lowest pitches of the trumpet. We also need to include in the table the known frequencies of these pitches. Now, using v = 33853 cm/sec, we calculate λ for each of the seven pitches. Finally, we compare λ_{max} to L by solving for u the equation $\lambda_{max} = uL$ for each of the seven values of λ which were just calculated. The completed table, then, is found on the following page.

Ideal results would have been obtained if the

1a	Examined Pitch Written:	F#3	G3	G#3	A3	Bb3	B3	C4
	Sounding:	E3	F3	F#3	G3	Ab3	A3	Bb3
1b	Known frequency of this pitch (hz):	164.81	174.61	185.00	196.00	207.65	220.00	233.08
	Length of pipe (L) (cm):	204.2	192.0	183.9	172.5	160.3	152.2	140.0
λ	(= 33853 cm/sec $\div$ f) (cm):	205.4	193.9	183.0	172.7	163.0	153.9	145.2
u	(= $\lambda_{max} \div$ L):	1.01	1.01	.995	1.00	1.02	1.01	1.04

value of u turned out identical for each examined pitch. The values obtained, however, range from .995 to 1.04 and average out to 1.014. Now it is more reasonable to assume that the average error in the calculations is .014 than to take the value of u to be precisely 1.014. If, therefore, we take λ_{max} to be equal to (1)L, we can test the equation $v = f\lambda$ by the same method that was used for the woodwind instruments. The results thus obtained are excellent, as shown in the table on the following page.

1a Examined Pitch Written:	F#3	G3	G#3	A3	Bb3	B3	C4
Sounding:	E3	F3	F#3	G3	Ab3	A3	Bb3
1b Known frequency of this pitch (hz):	164.81	174.61	185.00	196.00	207.65	220.00	233.08
Length of pipe (L) (cm):	204.2	192.0	183.9	172.5	160.3	152.2	140.0
λ (where λ_{max} = L) (cm):	204.2	192.0	183.9	172.5	160.3	152.2	140.0
Calculated Frequency (33853 cm/sec $\div$ L) (hz):	165.78	176.31	184.08	196.25	211.19	222.42	241.81
2a Pitch whose frequency in equal temperament is nearest to that obtained above:	E3	F3	F#3	G3	Ab3	A3	B3
Known frequency of this pitch (hz):	164.81	174.61	185.00	196.00	207.65	220.00	246.94
Discrepancy of 2a with respect to 1a (by interval):	none	none	none	none	none	none	- 2nd hghr.
Ratio of calculated frequency with respect to 1b:	1.01	1.01	.995	1.00	1.02	1.01	1.04

The French Horn

It is not possible, for acoustical reasons, to construct a single horn of either true treble or true bass range which has real horn tone quality.[18] For this reason, the horn in common use today is a double instrument, consisting essentially of the combination of a horn in F and a horn in B-flat. The same mouthpiece and bell are used for each horn by being switched through two separate channels of different lengths by means of a special valve operated by the left thumb. The three main valves each have two sets of valve ports and two sets of valve slides, one for each channel, so the same three valves are used for both horns. The length of the main air column in the F horn is extremely great -- about 370 cm; to keep its physical length to reasonable proportions, it is coiled into a circle.

Aside from the differences in timbre and range (which will be discussed later), there are two fundamental differences between the horn and the trumpet. First, while the trumpet rarely plays above the eighth resonance mode, the horn uses the series up to an octave higher, as far as the sixteenth.

Since partials which lie above the seventh are all a diatonic whole step or less apart, the problem of accuracy on the horn is somewhat more serious than on the trumpet. Use of partials as high as the sixteenth requires that the high resonances of the instrument be pronounced and distinct. The shape of the air column in the horn has been derived by experiment and experience to accomplish this; the leadpipe is in the form of a long cone with a gradual taper, the middlepipe is almost cylindrical, and the end flares out rapidly into a large bell.

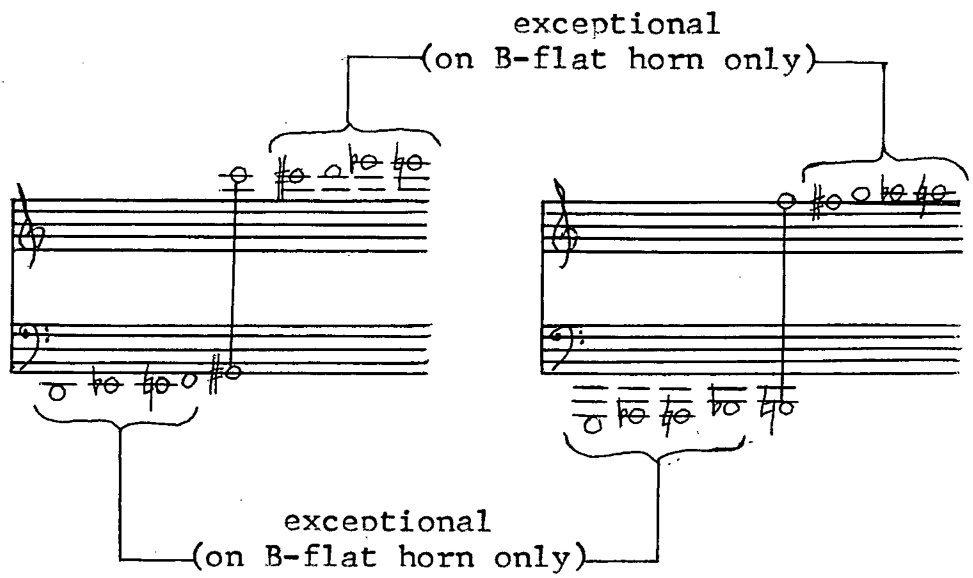
The second fundamental difference between the horn and the trumpet lies in the construction of the mouthpiece. The horn mouthpiece differs from that of the trumpet in that it has a deeper cup with a more gradual transition to the back bore. The acoustical reasons for this difference have still not been determined.

Since the horn plays in a range where the modes are close together (as will be shown in detail later), it has more available notes and thus more flexibility than, for example, the bugle. Nevertheless, it still gives only the notes which correspond to its modes. For this reason, the length of the horn was originally chosen to give a series of modes whose frequencies were in the key of the composition being played. This was done by using a single horn together with interchangeable pieces called crooks, which could be inserted at the mouthpiece end of the horn to add to the length of the instrument and put it in the desired key. This usually required the horn player to carry around with him several pounds of crooks.

The development of valves and their application to horns eased the horn player's problems tremendously, because it became possible to play in different keys without having to change crooks. The additional length of tubing brought into use by a valve plays the same role formerly played by a crook -- that of putting the horn into a different key. The principle -- that of inserting additional tubing -- is the same as for the trumpet, however, horn valves are constructed differently, being based on rotational rather than sliding action.[19]

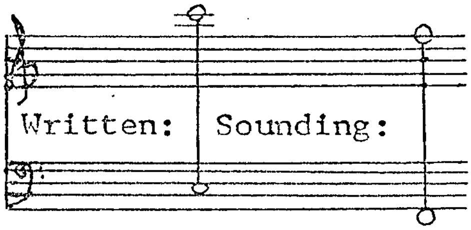
The working compass of the horn is as follows:

Written: Sounding:

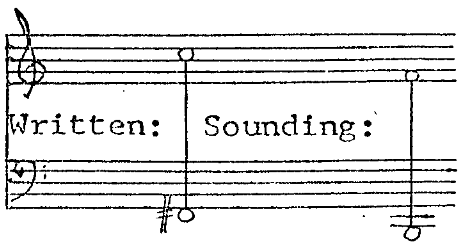


Although this range is wide on paper, in practice it is obtainable only by the most outstanding players, because of the difficulty which arises in forming an embouchure capable of producing pitches in both extremes with certainty and comfort. Consequently, orchestral players specialize in either the higher or lower part of the range. Generally, the first and third players play parts written in the upper part of the compass, and the second and fourth players play the lower parts. Between them, the two pairs of players divide the complete normal range into two overlapping portions, as shown on the following page:

Horns 1, 3:



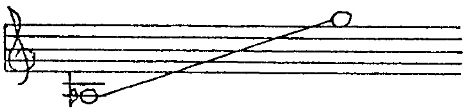
Horns 2, 4:



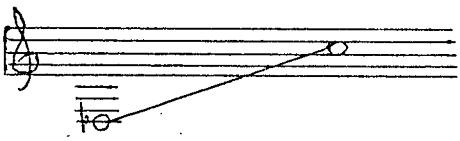
It goes without saying that these limits are not strictly applied, especially if, as is often the case, all four players are required to play in unison.

Like most instruments, the horn has its "comfortable" range, in which its tone is superior. This region is approximately from Ab3 to G5:

Written:

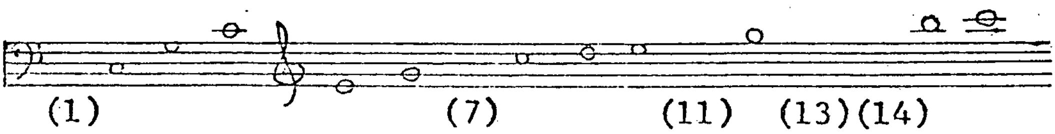


Sounding:



As will be shown later, this is the range of the eighth mode of oscillation of the two horns. Below the lower limit of this range the tone can easily become somewhat harsh and coarse. Above the upper limit the tone becomes bright, and a pianissimo is very hard to achieve there. In its medium register the horn is at its most characteristic timbre.[20]

Although the air column of the horn most resembles a cone, its great length of tubing makes the K factor very small. In fact, the tube is *almost* cylindrical, except for some flare in the leadpipe and at the opposite end beyond the point at which it begins to flare out into a bell. Consequently, the horn does not behave like an instrument belonging to either the cylindrical or the conical group. It does, however, lend itself easily to overblowing and to operation in up to 16 modes of oscillation, which correspond to the harmonic series. Of these 16, modes 7, 11, 13, and 14 are not used because the resulting pitches are not in tune in equal temperament. Furthermore, the first mode is not used because it is difficult to produce owing to the great length and narrow bore of the instrument. Thus we are left with 11 useable resulting pitches for each fundamental. The natural harmonic series of the F horn is based on written C2, sounding F1, a perfect fifth lower, and the useable members of its overtone series are the following:

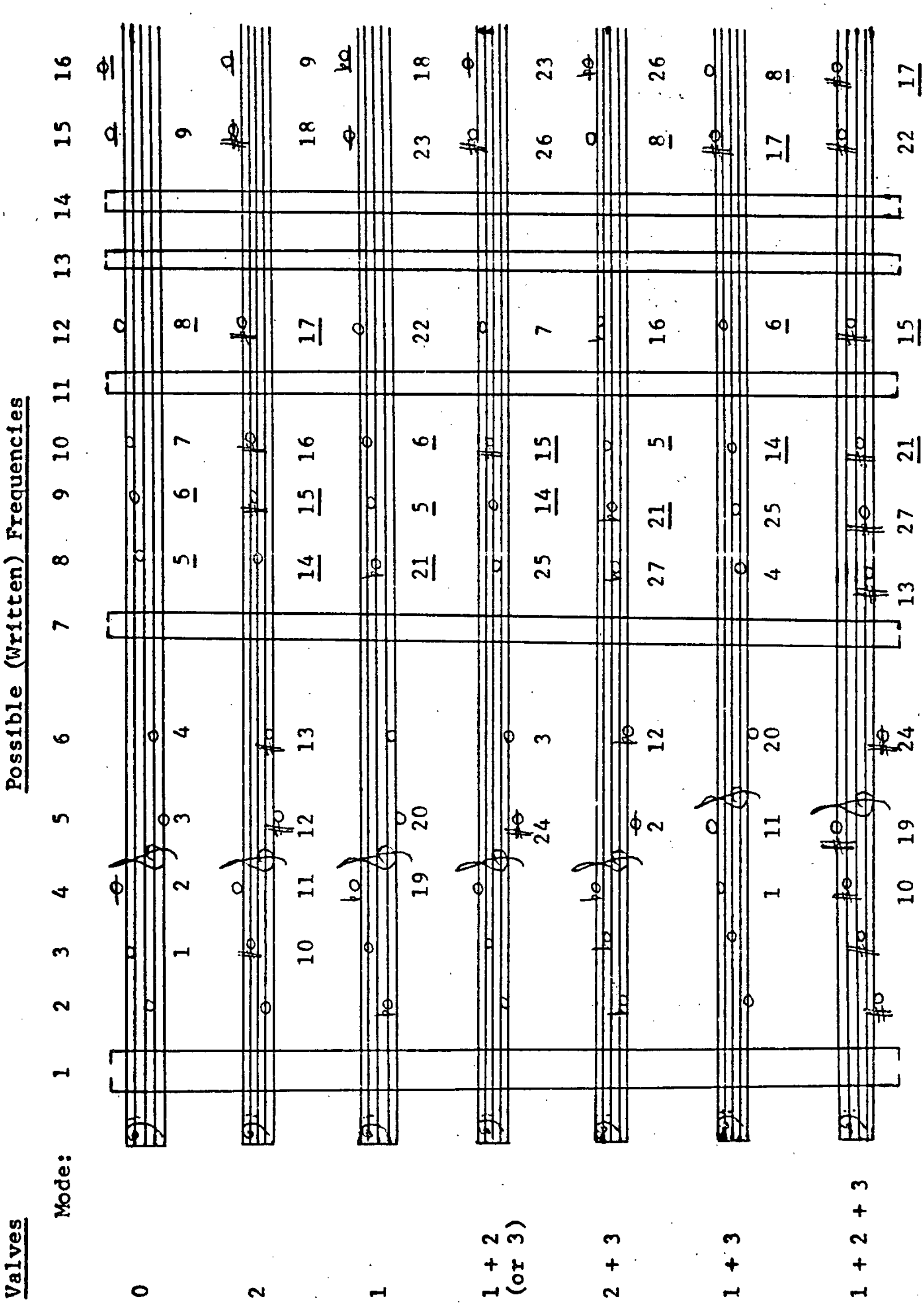


In order to provide the horn with a complete scale, the intervals between the notes shown above must be filled in. The largest interval which needs to be filled in is the first one, that of a perfect fifth, which requires six additional semitones to fill the space. These are provided by lengthening the air column through the use of valves. Six additional lengths are sufficient; in addition to filling in the space between the first two modes, they allow the horn to play six semitones below the lowest pitch shown above, and are more than adequate to fill in the gaps between the higher modes -- at least on paper.

As in the trumpet, three valves are used in the horn. They function in the same way as trumpet valves do, shifting the original frequency -- produced without any valves -- downward. The following table shows the resulting shift of frequency (in semitones) for each valve combination:

Valve(s)	Number of semitones
2	1
1	2
3	3
1 + 2	3
2 + 3	4
1 + 3	5
1 + 2 + 3	6

Through the use of these valves, the following pitches may be produced on the F horn as indicated:



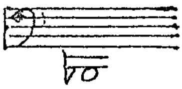
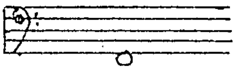
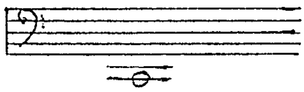
This is so because each fundamental which results from a new combination of valves has its own overtone series, the members of which are accessible in each case by overblowing.

Although there are seven rows containing eleven notes each, only 43 of these 77 notes are different. These 43 notes have been subdivided into three categories as follows:

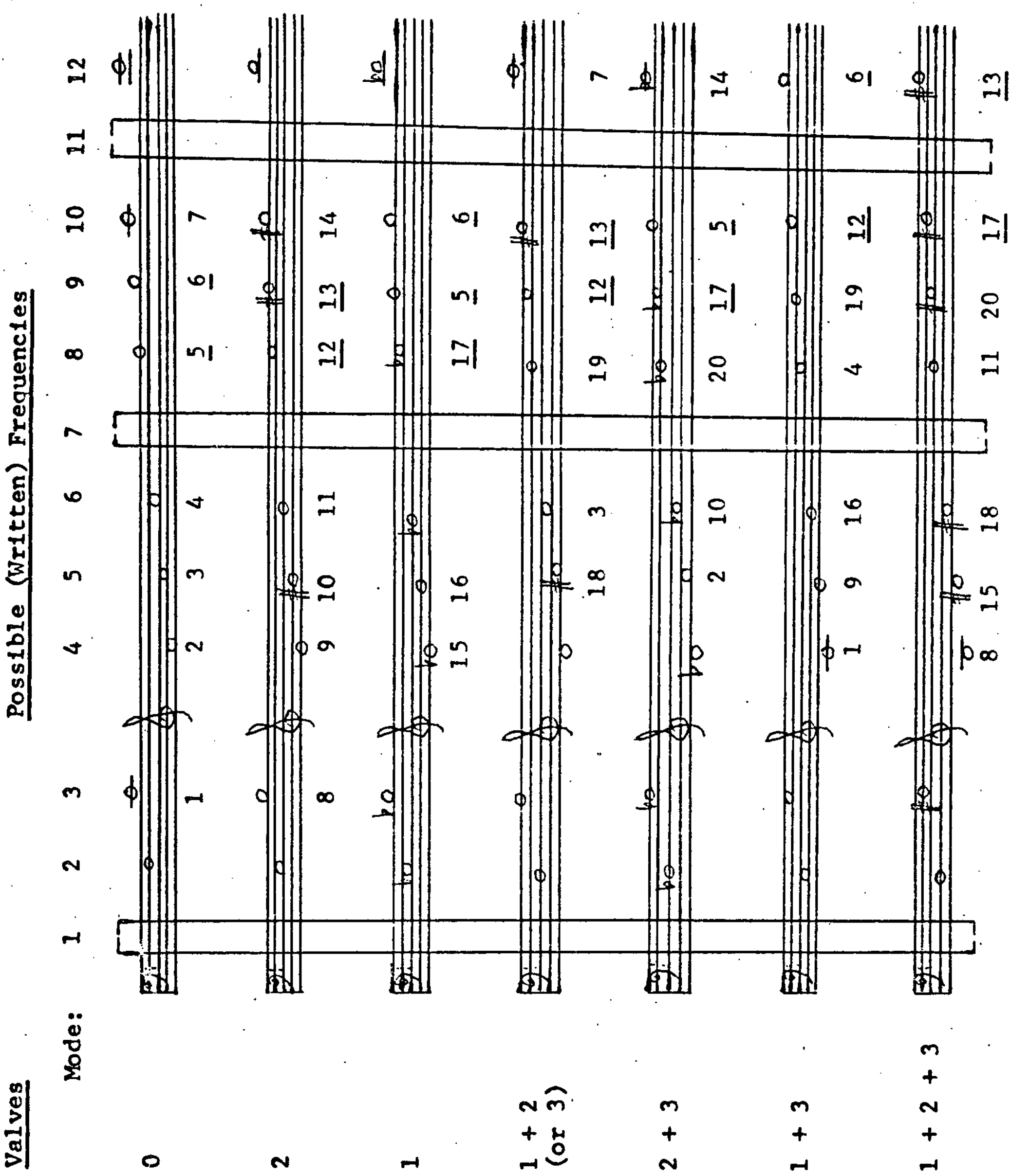
1) 16 pitches which appear only once -- unnumbered,
2) 20 pitches which appear twice each -- indicated with the same number in both places,
3) 7 pitches which appear three times each -- indicated with the same number underlined in all three places.

The next step is to determine which valves the horn player would, in practice, use if he wanted to produce these 43 pitches on the F side of his instrument. Clearly, for the 16 pitches for which only one fingering is possible he has no choice. These pitches are indicated in whole notes in the table on page 153. For the remaining 27 pitches, the process of elimination is used to determine which fingering will give the best result. Although the fifteenth mode of oscillation may be used, pitches produced by it are not as well in tune in equal temperament as pitches produced through modes 12 and 16. Since every note in this column may also be produced through the twelfth or sixteenth mode of oscillation, this entire column can be eliminated. This means that pitches 9, 18, 23, and 26 will be produced through the sixteenth mode, i. e. by overblowing at the triple octave, and pitches 8, 17, and 22 will be produced through the twelfth mode. For the 20 other remaining pitches, the one will be chosen which comes first, (reading from top to bottom), because this way all but five of these pitches will be produced through the use of one valve only. This is, in general, better than the use of two or more valves, because the pitches are produced as lower harmonics from a higher fundamental, rather than as higher harmonics from a lower fundamental which would be more difficult. All 27 pitches are indicated in half notes in the table on the next page. For pitches 6 and 15, the alternative use of the tenth mode of oscillation is also good; these alternatives are indicated in bracketed half notes.


Valves
Written Frequencies Produced
Mode: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
0
2
1
1 + 2 (or 3)
2 + 3
1 + 3
1 + 2 + 3

The B-flat side of the horn will now be examined. The natural harmonic series of this horn is based on sounding Bb1, a perfect fourth higher than the corresponding pitch of the horn in F. Ordinarily, for a B-flat instrument, this would be written as C2, a major second higher than it sounds. If, however, this notation were used for the B-flat side of the french horn, these pitches would sound a major second lower than written, while the remainder sound a perfect fifth lower than written. This problem is solved by transposing all pitches written for the B-flat side of the horn by a perfect fourth up -- as if they were also written for horn in F. Thus, the sounding pitch Bb1  would be notated as F2  rather than as C2 . This does not alter the fingerings used, since any given sounding pitch is fingered the same way regardless of where it is notated. The natural harmonic series of the horn, then, is based on written F2, and the highest useable member of its overtone series is the twelfth harmonic:

When we proceed, as before, to provide this horn with a complete scale by filling in all the intervals between the notes shown above, we obtain the following possible frequencies:



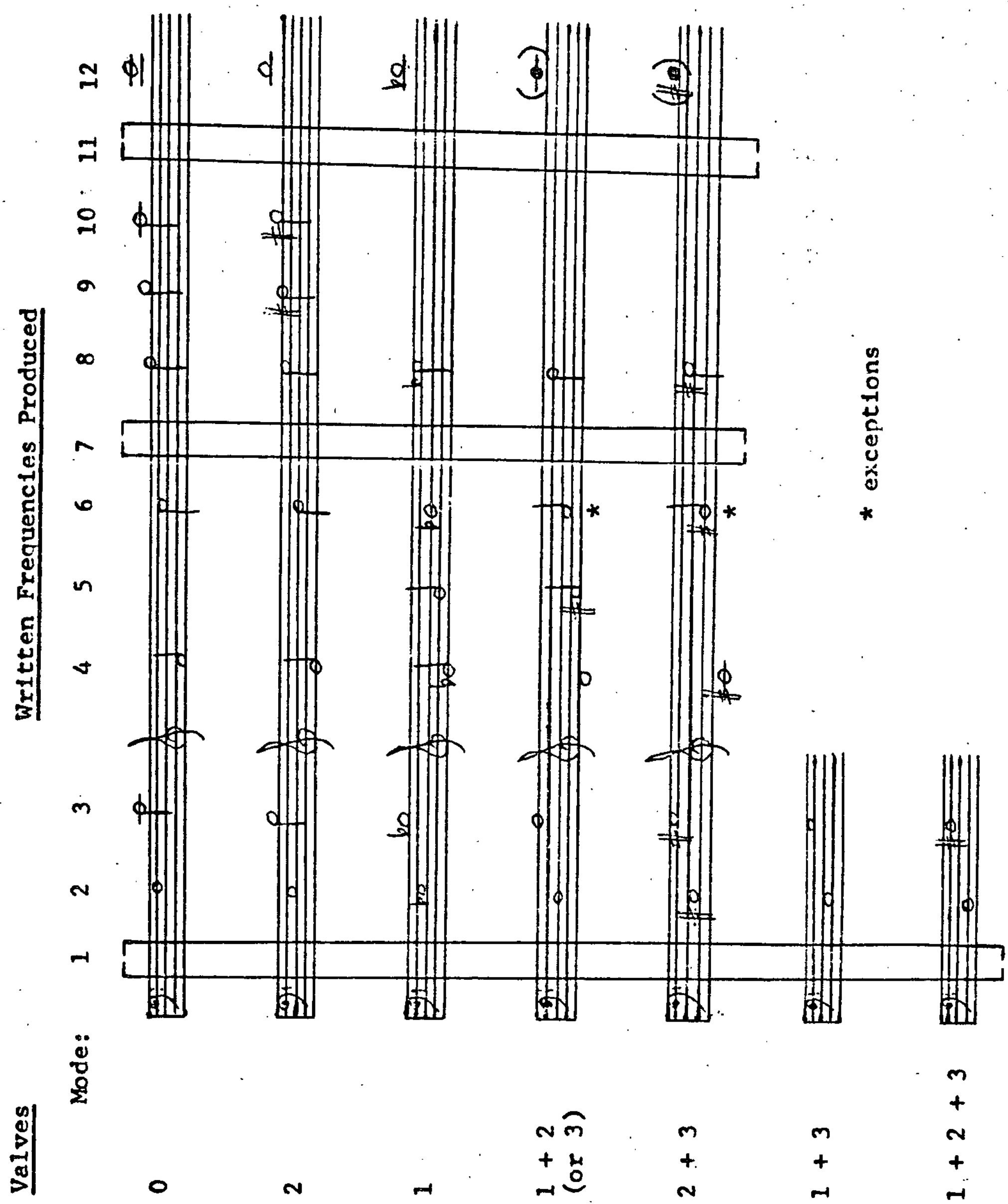
Although there are seven rows containing nine notes each, only 38 of these 63 notes are different. These 38 notes have been subdivided into three categories as follows:

1) 18 pitches which appear only once -- unnumbered,

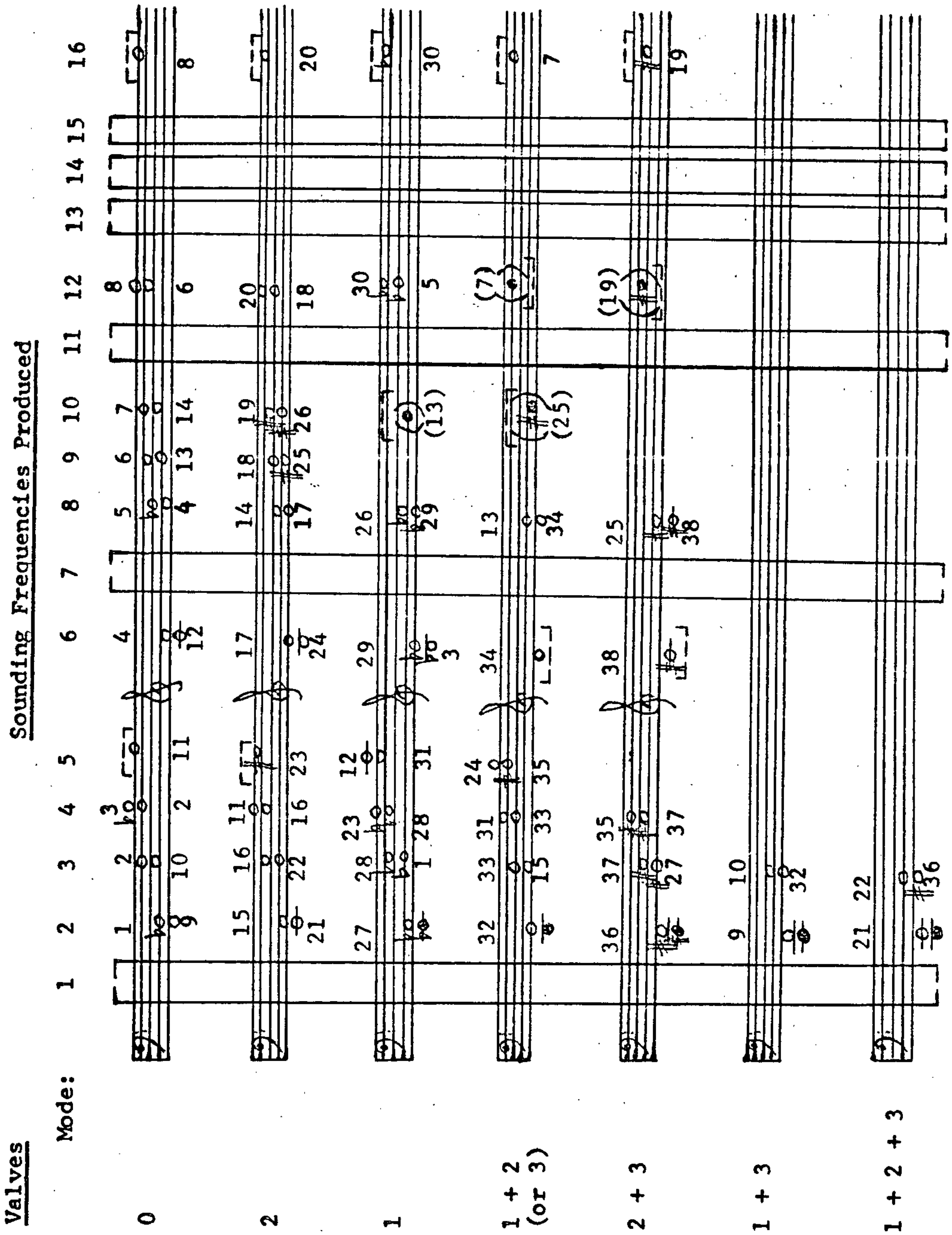
2) 15 pitches which appear twice each -- indicated with the same number in both places,

3) 5 pitches which appear three times each -- indicated with the same number underlined in all three places.

As before, the next step is to determine which valves the horn player would, in practice, use if he wanted to produce these 38 pitches on the B-flat side of his instrument. Clearly, for the 18 pitches for which only one fingering is possible he has no choice. These pitches are indicated in whole notes in the table on the following page. For the remaining 20 pitches, the one is chosen which comes first, reading from top to bottom, for the same reasons as before. For reasons to be shown later, exceptions will be made for pitches 3 and 10. The alternate fingerings will be used for these two pitches, although the fingerings which would normally be chosen may also be used. All 20 pitches are indicated in half notes in the table below. For pitches 7 and 14, the use of the twelfth mode is also good; these alternatives are indicated in bracketed quarter notes.



The final tables for each of the two horns may now be superimposed, showing the sounding pitches for each written pitch shown before. For each pair of notes, the top one is produced on the B-flat side of the horn; the bottom one is produced on the F side.



Although there are 81 notes, only 43 of them are different. These 43 notes have been subdivided into two categories as follows:

1) the 5 lowest pitches which appear only once -- indicated in quarter notes and unnumbered,

2) the remaining 38 pitches which appear twice each indicated in whole notes and with the same number in both places.

The above table shows that of the entire range of 43 notes of the whole horn, 38 can theoretically be produced on either horn. In practice, however, all the pitches of the lower half of the range are produced on the F horn, and those of the upper half on the horn in B-flat. This, of course, is the whole purpose of having two horns -- so that low harmonics are used rather than high ones.

Several more specific observations can be made from the table:

1) The use of the sixteenth mode of the horn in F is really never necessary; it only provides the player with optional fingerings for the top five pitches of the range of the instrument.

2) Only three of the pitches in the twelfth column

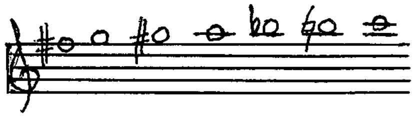
(pitches 8, 20, and 30) cannot be produced through a lower mode of oscillation.

3) Only two of the pitches in the tenth column (pitches 7 and 19) cannot be produced through a lower mode of oscillation.

4) Only two of the pitches in the ninth column (pitches 6 and 18) cannot be produced through a lower mode of oscillation.

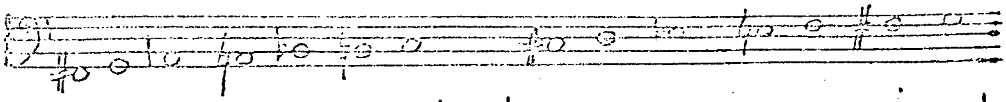
The conclusion to be reached from these observations is that the use of modes (of either horn) higher than the eighth is only necessary for the top 7 of the 43 pitches of the instrument's normal range:

Written:



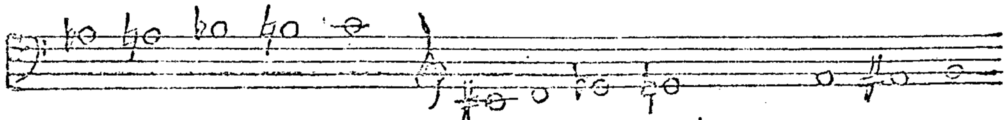
Sounding:

The complete chormatic scale of the entire horn may now be written out, showing the fingerings (by valve numbers) and the modes of oscillation used. All the notes are written as for horn in F, therefore all sound a perfect fifth lower than written. When the B-flat horn is to be used, this is indicated by the letter "T" for "thumb key".[21]



second — third

F:	1	1	2	1	1	2	0	1	1	2	1	1	2	0
	2	3	3	2				2	3	3	2			
	3							3						



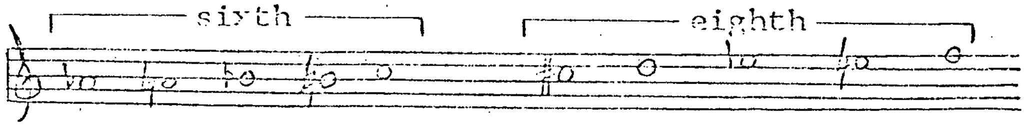
fourth — fifth — sixth

F:	2	1	1	2	0	1	1	2	0	1	2	0
	3	2				2						

2 0: Alternate (fifth mode); seldom used.

Bb:	2	1	1	2	0	2	1	1	2	0
	3	2	T	T	T	3	2	T	T	T
	T	T				T	T			

sixth — eighth



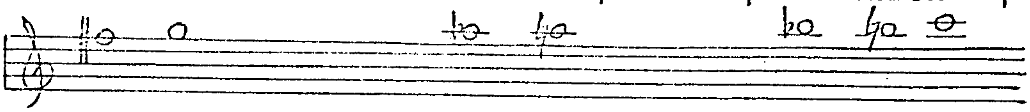
eighth

F:	2	1	1	2	0
	3	2			

2 1 : Alternate (twelfth
3 2   mode); seldom used.

Bb:	2	0	2	0	1	2	0
	T	T	T	T	T	T	T

ninth — tenth — twelfth



The overlap from Ab4 to C5 represents the recommended area for switching horns. This is why the alternate fingerings for the pitches Ab4 and A4 (originally numbered 3 and 10) were chosen. This way, the fingerings for these pitches are easy to remember because they are identical for the two horns. The regular fingerings may also be used, as may the other alternatives shown above. The choice is often made according to the key in which a piece of music is written; certain fingerings being superior to others in specific keys.

In practice, the side of the horn to be used to produce these five pitches depends on the context in which they appear. When approached from below, they are played on the F side, and when approached from above, they are played on the B-flat side. In certain cases, all pitches up to C5 may be produced exclusively on the F horn.

It may be added here, that it is one of the horn player's obligations to learn to match the sound of the two sides of his instrument to each other, especially in passages where changing sides is involved.

Use of the Right Hand

Prior to the development of valves, when crooks were still being used, it was discovered that if the horn were shaped so that the hand could be placed in the bell, the pitch of the instrument could be lowered by an amount that depended on how far the hand was inserted. Placing the hand in the horn bell restricts its area; this increases its acoustic mass at the end of the resonant air column, and lowers the frequencies of the horn. Furthermore, the hand acts as a reflector and sends the resonances back into the horn. It fixes the notes into position, which facilitates accuracy in playing by making the notes easier to land upon. The technique of hand closure was developed to the point where it was possible to play a complete chromatic scale over a part of the valveless instrument's range. However, the hand-closed tones differed in quality from the open tones, and much practice was required to get a reasonably even scale.[21]

With the invention of the valve, hand stopping for the purpose of obtaining chromatic notes became unnecessary but the practice was retained as a means of varying tone colour and facilitating playing. Nowadays, the horn is still played with the right hand in the bell, so that the pitch is always slightly flatter than the length of the tubing should produce. For this reason, horns are commonly built slightly sharp.[22]

Generally speaking, if the horn is played without the hand in the bell, its tone is somewhat coarse and brassy. Placing the hand in the bell has a refining effect on the tone, giving it a slightly veiled and quite characteristic colour unlike that of any other instrument.

The position of the hand in the bell should be, generally speaking, such as to allow almost unobstructed passage of the sound between the palm of the hand and the inner side of the bell. The exact position of the hand can only be determined by trial and error; almost everything depends upon the size of the hand, the bore of the instrument, and the tone quality which is desired. The more the bell is covered, the darker the tone will be, and the flatter the pitch will be. Conversely, the less the bell is covered, the brighter the tone will be, and the sharper the pitch.[23]

If the bell is closed off as much as possible with the hand, a tone of quite different quality, commonly called a stopped tone, is produced. The pitch apparently rises by about a semitone when this is done, so the player must transpose down. What happens during hand stopping is an extension of the hand-closure effect described above. The acoustic mass at the end of the horn is increased so much that the frequencies of all the modes are pulled down to where each mode has moved to about a semitone above the original frequency of the mode below it. A given note is then played in the next higher mode when the horn is stopped, the one originally used for the note having gone much too flat. This can be demonstrated by blowing a note on the horn while the right hand slowly closes off the bell and reaches stopping position. If no attempt is made to keep the note in tune by using the lips, the mode can be followed smoothly down in frequency to its final position, below its original value. If the player tries to hold the pitch, the note jumps abruptly to the next (higher) mode as the hand is inserted, and ends up higher in frequency than its original value. A given stopped note is thus played in a higher mode than the unstopped note.[24]

Testing the Equation $v = f\lambda$[25]

To test the equation $v = f\lambda$ in a practical situation involving the horn, the same method must be used that was used for the trumpet. Again, since the horn is partly cylindrical and partly conical, neither $\lambda_{max} = 2L$ nor $\lambda_{max} = 4L$ holds true. Consequently, we must substitute known values of v and f into the equation, which will yield λ, which we will then compare to L.

It might be pointed out here, that even if $\lambda_{max} = 2L$ or $\lambda_{max} = 4L$ were true, another problem would arise if we tried to adopt exactly the same procedure that was used for the woodwind instruments. As was the case with the other instruments of the conical bore family, we would here also have to extrapolate the cone to its theoretical peak. Since, however, we are dealing with two horns in one, we would have to make two extrapolations: one for each horn. This would result in two different values of length (previously denoted by r_1) and of volume for the missing portion of the cone, which is replaced by the mouthpiece. Consequently, we would conclude that a different mouthpiece must be used for each side of the horn -- the volume of each one being equivalent to the volume of the missing portion of the cone. Since we know that the same mouthpiece is used for both sides of the horn, we know that this approach would be incorrect simply because it is impossible for one mouthpiece to be of two different volumes.

We proceed now with the testing of the equation. It will be best to work on the two sides of the horn separately, treating them as individual instruments. Let us first test the horn in F. To begin with, we need to know the lengths of all the parts of the horn. These lengths were measured and found to be as follows:

Part	Length (in cm)
Mouthpiece	4.6
Leadpipe	81.7
Middlepipe	148.2
Bellpipe	124.0
Bell	14.0
Basic Length (equals total of above lengths)	372.5

Tubing added by valve 1	44.0
Tubing added by valve 2	19.0
Tubing added by valve 3	72.5
Tubing added by valves 1 + 2	63.0
Tubing added by valves 2 + 3	91.5
Tubing added by valves 1 + 3	116.5
Tubing added by valves 1 + 2 + 3	135.5

(The terms "leadpipe, middlepipe, and bellpipe" are used for convenience only. Actually, the tube is quite continuous, and contains many bends rather than consisting of three distinct sections.)

Although this has no direct application to the testing of the equation, one additional observation may be made at this point. The table above shows that the length of tubing added by valve 3 is actually greater than that added by valves 1 + 2. (This is also true of the horn in B-flat.) With the trumpet, the opposite was the case. This is because modification of the third valve slide of the horn must meet different requirements than the corresponding modification in the trumpet. Because the higher harmonics of the horn are used more, the third valve is used less often than in the trumpet.

Returning now to the experiment, from the values obtained by measurement, we may easily find, by referring back to the fingering chart on page 153 and by addition, the total length of the pipe (L) used to produce the 7 lowest pitches of the horn. We also need to include in our table the known frequencies of these pitches. Now, using v = 33853 cm/sec, we calculate λ for each of the 7 pitches. Finally, we compare λ_{max} to L by solving for w the equation $\lambda_{max} = wL$ for each of the 7 values of λ which were just calculated. The completed table, then, is found on page 171.

Before interpreting the results, let us test the B-flat side of the horn. Again, we start by measuring the lengths of all the parts of the horn.

Part	Length (in cm)
Mouthpiece	4.6
Leadpipe	81.7
Middlepipe	55.1
Bellpipe	124.0
Bell	14.0
Basic Length (equals total of above lengths)	279.4
Tubing added by valve 1	35.0
Tubing added by valve 2	15.0
Tubing added by valve 3	56.0
Tubing added by valves 1 + 2	50.0
Tubing added by valves 2 + 3	71.0
Tubing added by valves 1 + 3	91.0
Tubing added by valves 1 + 2 + 3	106.0

It was not possible to determine by observation the length of tubing cut out by the thumb key. With the basic length of the F horn being 372.5 cm, the basic length of the B-flat horn was assumed to be 372.5(3/4) = 279.4 cm, i. e. 93.1 cm less. Thus, the length of the middlepipe was estimated to be 148.2 - 93.1 = 55.1 cm.

From the values above, we again find, by referring to the fingering chart on page 157 and by addition, the total length of the pipe (L) used to produce the 7 lowest possible pitches of the B-flat side of the horn. (In practice these pitches are produced on the F side of the horn.) We also include in our table the known frequencies of these pitches. Now, using v = 33853 cm/sec, we calculate λ for each of the 7 pitches. Finally, we compare λ_{max} to L by solving for x the equation $\lambda_{max} = xL$ for each of the 7 values of λ which were just calculated. The completed table, then, is found on page 172.

Looking at the two tables together, we can make the following observations:

1) The values of w range from 1.04 to 1.08 and average out to 1.054; the values of x range from 1.04 to 1.07 and average out to 1.049. In each case,

Horn in F

		F#2	G2	Ab2	A2	Bb2	B2	C3
1a	Examined Pitch Written:	F#2	G2	Ab2	A2	Bb2	B2	C3
	Sounding:	B1	C2	Db2	D2	Eb2	E2	F2
1b	Known frequency of this pitch (hz):	61.735	65.406	69.296	73.416	77.782	82.407	87.307
	Length of pipe (L) (cm):	508.0	489.0	464.0	435.5	416.5	391.5	372.5
	λ (= 33853 cm/sec ÷ f) (cm):	548.4	517.6	488.5	461.1	435.2	410.8	387.7
	w (= λ_{max} ÷ L):	1.08	1.06	1.05	1.06	1.04	1.05	1.04

Horn in B-flat

1a	Examined Pitch							
	Written:	B2	C3	C#3	D3	Eb3	E3	F3
	Sounding:	E2	F2	F#2	G2	Ab2	A2	Bb2
1b	Known frequency of this pitch (hz):	82.407	87.307	92.499	97.999	103.83	110.00	116.54
	Length of pipe (L) (cm):	385.4	370.4	350.4	329.4	314.4	294.4	279.4
λ	(= 33853 cm/sec $\div$ f) (cm):	410.8	387.7	366.0	345.4	326.0	307.7	290.5
x	(= $\lambda_{max} \div$ L):	1.07	1.05	1.04	1.05	1.04	1.05	1.04

λ_{max} deviates from 1.00 the most for the lowest pitch -- the one whose production involves the use of all three valves. This is consistent with what was discussed about the nature of the valve system in the chapter on the trumpet. When all three valves are used together, the total length of tubing they add is insufficient for production of the desired pitch. This was sufficiently noticeable with the trumpet, but with the french horn it is even more noticeable, because greater lengths of tubing are involved. In practice, horn players move the third valve slide out with the little finger when using all three valves, and thus remove the discrepancy. In the calculations, however, nothing more was added to the basic length than the amount of tubing added by the three valves when used individually. This showed up very clearly in the results.

2) In view of the above, if we exclude from consideration the lowest pitch of each of the horns, the values of w and x average out to 1.05 and 1.045 respectively. If we want to claim that, in reality, λ_{max} = (1)L in both cases, these deviations must

be accounted for. In fact, the discrepancy evident in these results was already predicted on page 164, where the use of the right hand was discussed. Generally, in any open tube, the terminal antinode is formed slightly beyond the open end, which makes the tube's acoustical length slightly longer than its physical length. This phenomenon was already encountered with the second instrument which was tested, the flute. The difference between the two lengths, known as the end correction, is especially great when the open end is obstructed; this is exactly the case when the hand is inserted into the bell. For the horn in F, the end correction has turned out to be 5% of the basic length, i. e. 18.6 cm, and for the horn in B-flat, it appears to be 4.5% of the basic length, i. e. 12.5 cm. Thus it seems that the longer the tube, the greater percentage of its basic length must be added as end correction to determine the actual acoustical length of the tube. Clearly, the small amount of testing which has been done here is insufficient for making general conclusions. The difference between the two values (.5%) could well be attributable to nothing but experimental error.

In any event, if we take λ_{max} to be equal to (1)L plus end correction, we can test the equation $v = f\lambda$ by the same method that was used for the woodwind instruments. The results thus obtained are excellent for both horns, as shown by the tables on the following two pages.

Horn in F

1a Examined Pitch Written:	F#2	G2	Ab2	A2	Bb2	B2	C3
Sounding:	B1	C2	Db2	D2	Eb2	E2	F2
1b Known frequency of this pitch (hz):	61.735	65.406	69.296	73.416	77.782	82.407	87.307
Length of pipe plus end correction (L+) (cm):	526.6	507.6	482.6	454.1	435.1	410.1	391.1
λ (where λ_{max} = L+)(cm):	526.6	507.6	482.6	454.1	435.1	410.1	391.1
Calculated frequency (33853 cm/sec ÷ L+) (hz):	64.286	66.692	70.147	74.550	77.805	82.548	86.558
2a Pitch whose frequency in equal temperament is nearest to that obtained above:	C2	C2	Db2	D2	Eb2	E2	F2
Known frequency of this pitch (hz):	65.406	65.406	69.296	73.416	77.782	82.407	87.307
Discrepancy of 2a with respect to 1a (by interval):	- 2nd hghr.	none	none	none	none	none	none
Ratio of calculated frequency with respect to 1b:	1.04	1.02	1.01	1.02	1.00	1.00	.991

Horn in B-flat

1a Examined Pitch Written:	B2	C3	C#3	D3	Eb3	E3	F3
Sounding:	E2	F2	F#2	G2	Ab2	A2	Bb2
1b Known frequency of this pitch (hz):	82.407	87.307	92.499	97.999	103.83	110.00	116.54
Length of pipe plus end correction (L+) (cm):	397.9	382.9	362.9	341.9	326.9	306.9	291.9
λ (where λ_{max} = L+)(cm):	397.9	382.9	362.9	341.9	326.9	306.9	291.9
Calculated frequency (33853 cm/sec ÷ L+) (hz):	85.079	88.412	93.285	99.014	103.56	110.31	115.98
2a Pitch whose frequency in equal temperament is nearest to that obtained above:	F2	F2	F#2	G2	Ab2	A2	Bb2
Known frequency of this pitch (hz):	87.307	87.307	92.499	97.999	103.83	110.00	116.54
Discrepancy of 2a with respect to 1a (by interval):	- 2nd hghr.	none	none	none	none	none	none
Ratio of calculated frequency with respect to 1b:	1.03	1.01	1.01	1.01	.997	1.00	.995

The Trombone

The body of the modern slide trombone consists of two straight cylindrical tubes, the mouthpipe and the middlepipe, plus a conical bellpipe which flares out much more rapidly than does, for example, the bellpipe of the trumpet. The three sections are parallel to each other and are connected by two semicircular bends. The first of these two bends is cylindrical, but the modern bell-bow is conical, so that the expansion of the bell actually begins somewhere in the bend. Sometimes the bell and the bow are manufactured as one piece. The greater part of the mouthpipe and the middlepipe form the inner of two pairs of closely fitting telescopic tubes, the outer pair of which is connected by the other bend which results in the U-shaped slide. The slide is freely movable, and by pulling it out, the overall length of the body may be extended by nearly two-thirds.[26]

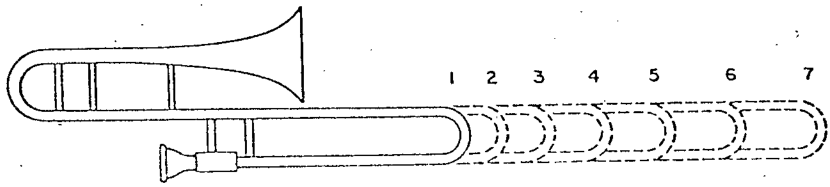
The family of slide trombones in use today consists of five instruments of various sizes: the Soprano, normally in B-flat (practically nonexistent), the Alto, normally in E-flat or F (rare), the Tenor in B-flat, the Bass in B-flat (convertible to F;

sometimes called the Tenor-Bass Trombone), and the Contrabass, normally in B-flat. All members of the slide trombone family are constructed in basically the same way -- the structural difference is mainly one of size. The bore diameters of modern Tenor Trombones vary from 1.27 cm to 1.39 cm; that of the Bass Trombone is approximately 1.42 cm. Because of the great similarity of the five instruments to each other, only one will be considered here: the Tenor in B-flat.

Although two-thirds of the trombone bore is cylindrical, the trombone does not behave like an instrument belonging to the cylindrical group. It does, however, lend itself easily to overblowing and to operation in at least 12 -- and in the hands of good players up to 16 -- modes of oscillation. Furthermore, the successive modes of oscillation coincide with the complete harmonic series. For practical purposes, it is best to take the twelfth harmonic as the last accessible one. Consequently, the trombone, as described thus far, is naturally capable of producing the following pitches:

Written <u>and</u> Sounding:

In order to provide the trombone with a complete scale, the intervals between the notes shown above must be filled in. For the moment, we will not try to fill in the notes forming the first (and widest) interval of the octave, since this would require 11 additional semitones to fill the space. Instead, we will aim to fill in the next interval of a perfect fifth, which requires only 6 additional semitones. This is accomplished by lengthening the air column by pulling the slide out an appropriate distance. Six additional lengths obtained in this way will be sufficient for the present goal; in addition to filling in the space between the second and third harmonics, they allow the trombone to play six semitones below the second harmonic, and are more than adequate to fill in the gaps between the higher modes. The trombone slide, then, can be placed in seven different useable positions, as shown on the next page: [27]



Playing positions of the trombone.

Through the use of these positions, the pitches shown in the table on the following page may be produced as indicated. This is so because each fundamental which results from a new slide position has its own overtone series, the members of which are accessible in each case by overblowing.

Although there are seven rows containing twelve notes each, only 45 of these 84 notes are different. These 45 have been subdivided into four categories as follows:

1) 23 pitches which appear only once -- unnumbered,

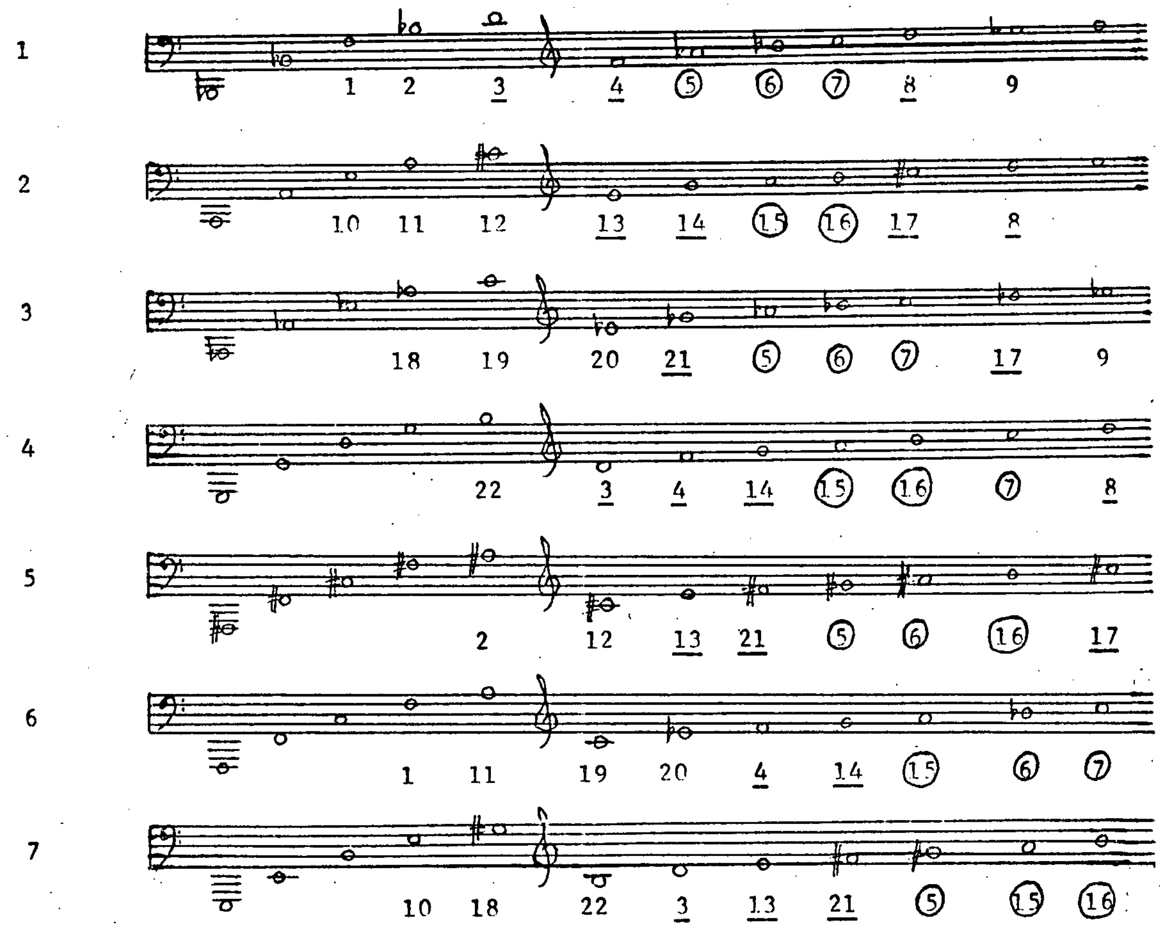
2) 10 pitches which appear twice each -- indicated with the same number in both places,

3) 7 pitches which appear three times each -- indicated with the same number underlined in all three places,

4) 5 pitches which appear four times each -- indicated with the same number circled in all four places.

Position | Possible Written (and Sounding) Frequencies

Mode: 1 2 3 4 5 6 7 8 9 10 11 12



The next step is to determine which positions the trombone player will, in practice, use to produce each of these 45 pitches of the range of his instrument. Clearly, for the 23 pitches for which only one position is possible he has no choice. For the remaining 22 pitches, the process of elimination is used to determine which positions will give the best result. In general -- although not always -- the shortest available position, i. e. the one which comes first, (reading from top to bottom) will be chosen. This is because a trombonist usually prefers to use the shorter positions rather than the longer ones, for reasons to be shown later. Consequently, we begin by eliminating all "third choice" and "fourth choice" positions for the 12 pitches for which these choices are theoretically available.

While it is true that the trombone player may very accurately adjust the tube length for any pitch he wishes to produce, it is also true that for seventh and eleventh harmonics the normal positions will always yield pitches which will deviate slightly from the ones desired, and shortening of the tube will always be necessary. Since it is physically impossible to shorten first position, the highest pitches of this position in these two modes (Ab4 and Eb5) are automatically eliminated. Furthermore, since the other pitches of mode 11 are easily obtainable in other modes, this entire mode is eliminated.

The eliminations which have been made yield the following results:

1) The 23 pitches which originally appeared only once are unaffected.

2) Only one choice of position remains for pitches 5, 8, 9, and 17.

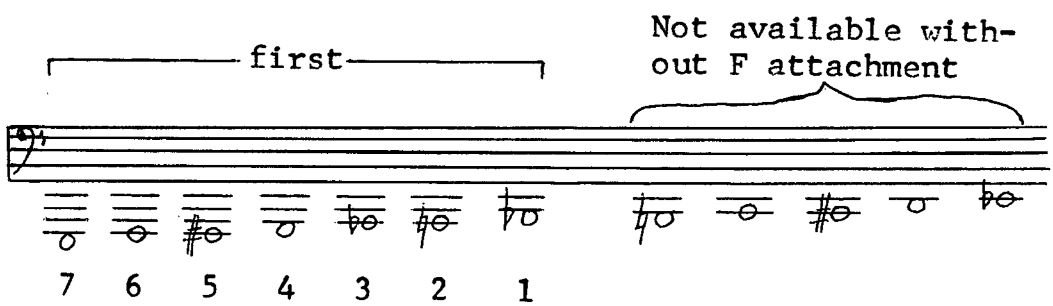
3) Two choices of position are left for the remaining 18 pitches of the trombone range: pitches 1, 2, 3, 4, 6, 7, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, and 22. In the table on the following page, the most common choice for each of these pitches is indicated with a quarter note, and the second choice with a bracketed quarter note.

Although "third choice" and "fourth choice" positions have not been included in the table on page 185, they do exist, and should not be totally ruled out. Some trombonists, especially jazz players, do make use of these positions. At any rate, Position — Written (and Sounding) Frequencies Produced

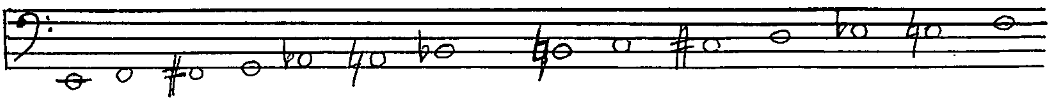
it is clear from this table that the use of positions 6 and 7 is unavoidable for production of only 6 of the 45 pitches of the range of the trombone.

It was decided previously not to begin by trying to fill in the 11 semitones between the first two harmonics, Bb1 and Bb2. As is now evident, 6 of these have meanwhile been filled in through the 6 additional positions which have just been examined. Thus, only 5 semitones -- B1, C2, C#2, D2, and Eb2 -- still need to be filled in. These 5 semitones are not obtainable unless the instrument is fitted with what is called an "F attachment". This device will be discussed further in a separate section.

The chromatic scale of the tenor trombone may now be written out, showing the positions and the modes of oscillation used.[28]



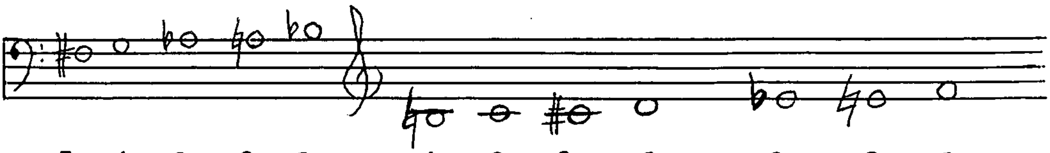
second — third



7 6 5 4 3 2 1   7 6 5 4 3 2 1

Alternate: (fourth mode): 7 6

fourth — fifth — sixth



5 4 3 2 1   4 3 2 1   3 2 1

Alternates: 7 6 5 (fifth mode)   7 6 5 4 (sixth mode)   6 5 4 (seventh mode)

seventh — eighth — ninth — tenth — twelfth

3 2   3 2 1   2 1   2 1   3 2 1

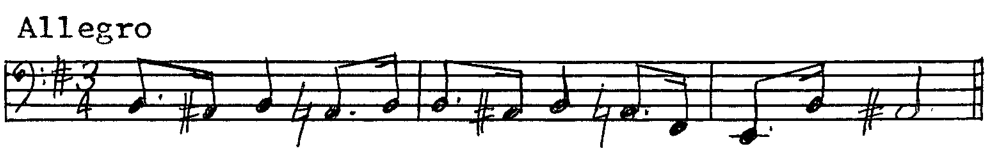
Alternates:

5 4 (eighth mode)   4 3 (ninth mode)   4 3 (tenth mode)

The Slide

The use of a slide is the simplest method available for lengthening a tube. A slide, however, demands that the tube be perfectly cylindrical for the entire length over which it is to operate. The trombone is the only instrument which uses this system. As was shown by the diagram on page 181, the slide arrangement of the trombone provides six additional lengths to give the additional six semitones. Going down a semitone amounts to approximately a 6 percent decrease in frequency, corresponding to about a 6 percent increase in length. Since the total length of the tube increases as the scale is descended, so does the 6 percent of the total length which must be added in order to descend another semitone. Consequently, each additional increase in length is slightly greater than the previous one, as the diagram on page 181 shows. The trombonist learns the precise locations of these positions through thousands of repetitions.[29]

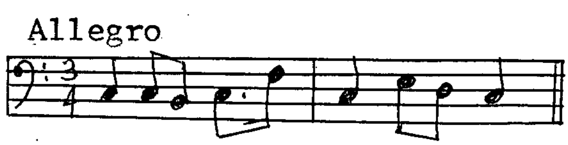
The slide system of changing tube length has one basic disadvantage. While a shift of one or two positions may be made quickly and accurately enough, a shift of five or six positions is hard to execute quickly and accurately, and takes up an amount of time which is not negligible. In this respect the valve system is clearly the superior one, because valve action is instantaneous, so the time taken up in depressing valves is negligible. In the case of the trombone, one major difficulty arises as a consequence of the nature of the system: many passages become practically impossible to play because of the awkward changes in positions involved. The following example shows one such passage which trombonists would rather not see.[30]



7 1 7 2 6 7 1 7 2 5 7------1

The conclusion to be drawn is that the trombone is at its best in the lowest positions. Thus, not only are wide shifts (such as 1 to 6 or 7) eliminated, but also the distances between adjacent positions are shorter. As can be seen from the diagram on page 181, the distance between positions 1 and 2 is smaller than that between 6 and 7, so the shift takes proportionately less time. Aside from this, production of sound is easier from a shorter tube than from a longer one. Furthermore, sounds produced as lower-numbered partials of fundamentals based on a shorter tube length are somewhat brighter in colour than the corresponding higher-numbered partials from a longer tube. This is why, in determining which position is generally used to produce any given pitch, the shortest available position was chosen.

Exceptions, however, do occur. In a passage in which positions 5, 6, and 7 predominate by necessity, if one note appears which may be produced using a high or a low position, the trombonist would choose the high position, simply because of convenience. In the following passage, for example, F3 could be produced in first position, but because of the context, sixth position would be the better choice.



6 6 7 6 6 6 2 4 6

(1)

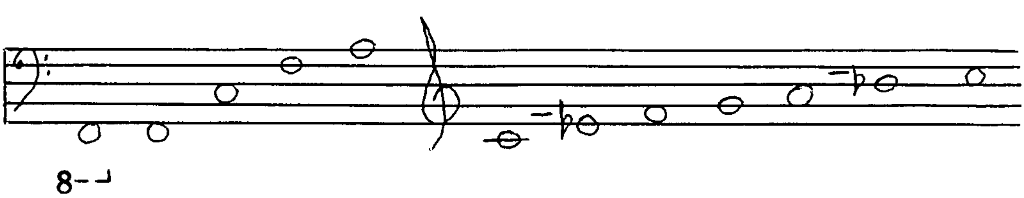
In general, because of the greater length of tubing involved, a richer tone results when a pitch is produced in a higher position.

It goes without saying that the trombonist should be aware of all the positions in which the pitches of the range of his instrument may be produced, and have these at his disposal as practical alternatives in special situations.

The F Attachment

The F attachment is an extra piece of tubing which is built in at the bend connecting the bellpipe to the middlepipe. It is activated by a trigger using a rotary valve operated by the left thumb. When this is done, the result is similar to that obtained by using the thumb key on the french horn: the instrument becomes a trombone in F. Consequently, the pitches which are available in first position are the following:

Written <u>and</u> Sounding:



With the extra length of tubing having been added, the distances between adjacent slide positions are greater than they were before. Accordingly, the slide is now only long enough to accomodate six positions, of which only the first is in the same location as that of the tenor trombone. The remaining five positions relate to the original locations as follows:[31]

Tenor-Bass Trombone	Tenor Trombone
2	♭2
3	♭3
4	#5
5	♭6
6	♭7

As was done previously, the entire table of theoretically possible frequencies is presented on the next page. In examining this table, we are interested primarily in:

1) The six pitches which are obtainable on the tenor trombone only in positions 6 or 7,

2) Any new pitches which are not obtainable on the tenor trombone. By comparing the pitches in this table to those in the table on page 185, we see that all the pitches originally produced through positions 6 and 7 are now available in positions 1 and ♭2 re-

Position With Trigger

Possible Written (and Sounding) Frequencies With F Attachment

Mode: 1 2 3 4 5 6 7 8 9 10 11 12

Positions: 1, ♭2, ♭3, #5, ♭6, ♭7

spectively.

Regarding 1) above, the six pitches in question are indicated in half notes in the above table. (The seven alternatives indicated in bracketed notes in positions 6 and 7 in the table on page 185 may now be regarded as alternatives in positions 1 and ♭2 respectively, although in practice they would be used in this way very rarely.)

Regarding 2) above, we find that eight new pitches appear: four of the five which formed the "gap" between the first two positions of the tenor trombone, and the lower octaves of these four pitches. These eight pitches are indicated in quarter notes in the above table.

A word of explanation is in order regarding the only remaining pitch from the original "gap" which is still not obtainable: B1. This is due directly to the fact that the seventh position has been lost because of the great lengths of tubing involved. A further consequence of this factor is that the low C's (C1 and C2) are almost certain to be too sharp, because the slide is not even long enough for a true sixth position. In practice, it is usually possible to pull the tuning slide out of the attachment to provide the extra length necessary for production of these three pitches. The other alternative is to provide the instrument with a second attachment, which is operated by a second trigger. This is commonly found on bass trombones, but not on tenor trombones.[32]

As can be seen from the table above, the F attachment provides more alternative positions for many pitches other than those which have already been discussed. If the trombonist is aware of all the alternatives the F attachment brings with it, he will have these at his disposal in special situations.

Just how valuable a resource the F attachment actually is can be clearly demonstrated by quoting once more the passage given on page 189. With the use of the F attachment, the passage becomes very simple to play:[33]

Allegro

♭2 1 ♭2 2 1 ♭2 1 ♭2 2 5 ♭2------1
T T T T T T

Testing the Equation $v = f\lambda$ [34]

As was done with the trumpet and the horn, we now wish to test the equation $v = f\lambda$ in a practical situation involving the trombone. As mentioned previously, the trombone tube is cylindrical for about two-thirds of its length. The proportion of cylindrical to conical length, however, increases with the positions used. All this means that the trombone does not behave like an instrument belonging to either the cylindrical or the conical group. Consequently, we cannot use $\lambda_{max} = 2L$ or $\lambda_{max} = 4L$, and must again substitute known values of f and v into the equation, which will give us λ, which we will then compare to L.

The parts of a trombone in first position were measured and found to be as follows:

Part	Length (in cm)
Mouthpiece	5.5
First Slide Stock	7.2
Slide	142.3
Second Slide Stock	8.2
Bellpipe	113.8
Basic Length (equals total of above lengths)	277.0

Now we need only to measure the amount of tubing added by each subsequent position of the slide to find, by addition, the total length of the tube for each position.

Position	Length of tubing Added to Basic Length	Total Length of Pipe (cm)
2	10.0	287.0
3	29.6	306.6
4	49.6	326.6
5	67.2	344.2
6	90.8	367.8
7	110.2	387.2

Referring back to the table on page 185, we may find the seven lowest pitches of the trombone (called the pedal notes) which correspond to the seven positions. We also include in our table the known frequencies of these pitches. Now, using v = 33853 cm/sec, we calculate λ for each of the seven pitches. Finally, we compare λ_{max} to L by solving for y the equation $\lambda_{max} = yL$ for each of the seven values of λ which were just calculated. The completed table, then, is found on the following page.

Ideal results would have been obtained if the value of y turned out to be exactly 2.0 for each examined pitch. In each case, however, the value of y is 2.1. If we want to claim that, in reality, $\lambda_{max} = 2L$, this deviation must be accounted for. By comparison to the corresponding deviations for

		E1	F1	F#1	G1	Ab1	A1	Bb1
1a	Examined Pitch:	E1	F1	F#1	G1	Ab1	A1	Bb1
1b	Known frequency of this pitch (hz):	41.203	43.654	46.249	48.999	51.913	55.00	58.270
	Length of pipe (L) (cm):	387.2	367.8	344.2	326.6	306.6	287.0	277.0
	λ (= 33853 cm/sec $\div$ f) (cm):	821.6	775.5	732.0	690.9	652.1	615.5	581.0
	y (= $\lambda_{max} \div$ L):	2.1	2.1	2.1	2.1	2.1	2.1	2.1

the two sides of the french horn, we see that this deviation is nearly identical:

Horn in F (average discrepancy of w)	5.0%
Horn in B-flat (average discrepancy of x)	4.5%
Tenor Trombone (average discrepancy of y)	5.0%

Consequently, it is most reasonable to assume that this discrepancy is attributable to end correction (and experimental error), as was the case with the flute and the french horn.

If we take λ_{max} to be equal to 2L plus 5% end correction, we can test the equation $v = f\lambda$ by the same method that was used for the woodwind instruments. The results thus obtained are excellent, as shown by the table on the following page.

1a Examined Pitch:	E1	F1	F#1	G1	Ab1	A1	Bb1
1b Known frequency of this pitch (hz):	41.203	43.654	46.249	48.999	51.913	55.000	58.270
Length of pipe plus end correction (L+) (cm):	406.6	386.2	361.4	342.9	321.9	301.4	290.1
λ (where λ_{max} = 2L+)(cm):	813.2	772.4	722.8	685.8	643.8	602.8	580.2
Calculated frequency (33853 cm/sec ÷ 2L+) (hz):	41.630	43.828	46.836	49.363	52.583	56.160	58.347
2a Pitch whose frequency in equal temperament is nearest to that obtained above:	E1	F1	F#1	G1	Ab1	A1	Bb1
Known frequency of this pitch (hz):	41.203	43.654	46.249	48.999	51.913	55.000	58.270
Discrepancy of 2a with respect to 1a (by interval):	none	none	none	none	none	none	none
Ratio of calculated frequency with respect to 1b:	1.01	1.00	1.01	1.01	1.01	1.02	1.00

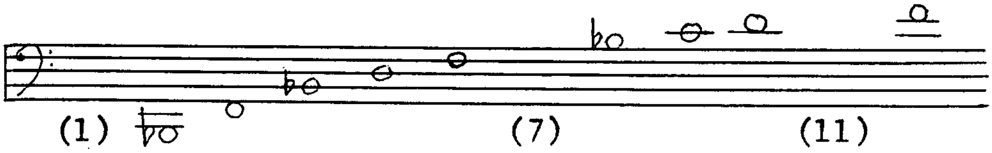
The Tuba

The tuba is the lowest-lying member of the brass family. Since it is capable of producing pitches which are even lower than A0 (27.5 hz), the lowest note of a piano, its basic length is extremely great -- about 550 cm. To reduce it to manageable proportions it is coiled four times around. Although the tube is actually quite continuous, for convenience in description, it may be considered to consist of four sections: the leadpipe, middlepipe, bellpipe, and bell. The middlepipe by itself is very short, but is lengthened through the use of valves, which add tubing to the basic length. This section is largely cylindrical; the other three sections are conical, but their K factor is not constant throughout.

The two most common tubas in use today are in BB-flat and C. (Two less common models are in E-flat and F.) The BB-flat tuba is used in bands and opera orchestras; the C tuba is the symphonic instrument on this continent. For consistency with the other brass instruments, only the BB-flat tuba will be considered here.

Since the bore of the tuba, like that of all brass instruments, is neither entirely cylindrical nor entirely conical, the tuba does not behave like an instrument belonging to either the cylindrical or the conical group. It does, however, lend itself easily to overblowing and to operation in at least 12 modes of oscillation. Of these 12, modes 7 and 11 are not used because the resulting pitches are not in tune in equal temperament. Furthermore, the entire first mode is seldom used because it is difficult to produce owing to the low frequency of its pitches and to the great length of the instrument. Thus we are left with nine useable resulting pitches for each fundamental. The natural harmonic series of the BB-flat tuba is based on Bb0, and the useable members of its overtone series are the following:

Written <u>and</u> Sounding:



In order to provide the tuba with a complete scale, the intervals between the notes shown above must be filled in. The largest interval which needs to be filled in is the first one, that of a perfect fifth, which requires six additional semitones to fill the space. These are provided by lengthening the air column through the use of valves.

Thus far, the situation described is directly comparable to that encountered with the trumpet and french horn. At this point, however, a fundamental difference arises. While with the trumpet and french horn three valves were sufficient to obtain all the desired pitches, this is not so easy with the tuba. It is theoretically possible with the use of the valve slide on the third valve, but even then certain valve combinations result in pitches which are sharper than the ones desired. For this reason, a fourth valve is always found on professional models. The necessity of the fourth valve is due directly to the deficiency inherent in the valve system, which was discussed on pages 131-136. Just as in the case of the french horn the valve combination 1 + 2, when used in combination with other valves, does not add enough tubing to lower the pitch by three semitones, thus necessitating the use of valve 3, similarly with the tuba the valve combination 1 + 3, even when used alone, does not add enough tubing to lower the pitch by five semitones, thus necessitating the construction and use of a fourth valve. This is because in the low range of the tuba the (physical, not proportional) length of the tubing which must be added every time a pitch is to be lowered is so great that the same valves which individually serve their purpose well fail to do so in combination. For the same reason a fifth valve is often added to the tuba. If used in the middle or high register (where it is seldom used), it would lower a given pitch by approximately 2½ semitones. When used in the low register, it lowers a given pitch by two semitones. This system is made workable by slide adjustment, which is fairly common even during playing.

The following table shows the resulting shift of frequency downward (in semitones) for each valve combination. (Combinations involving the fifth valve are not considered, because this valve is designed to be used only in the low register, where shifts totaling more than six semitones are made.)

Valve(s)	Number of semitones
2	1
1	2
3	3
1 + 2	3
2 + 3	4
4	5
1 + 3*	(5)
2 + 4	6
1 + 2 + 3*	(6)

The combinations marked * are almost never used because valve 4 is substituted for the combination 1 + 3.

With five valves at his disposal, the tuba player can descend more than 6 semitones below the pitch sounded with no valves. The following valve combinations have been derived by experiment and experience for each successive shift of frequency downward (in semitones):

Valves	Number of semitones
4 + 5	7
2 + 3 + 4	8
1 + 3 + 4	9
2 + 3 + 4 + 5	10
1 + 2 + 3 + 4 + 5	11

In comparing the two tables above, several observations may be made. If the fifth valve is taken to lower a pitch by exactly two semitones, then, according to the first table:

1) "7 semitones" follows from the combination "4 + T" because "5 semitones" follows from "4" alone, and "6 semitones" follows from "2 + 4". There is no conflicting situation here.

2) "8 semitones" does not follow from "2 + 3 + 4", because 1 + 3 + 5 = 9 semitones, and because "2 + 4" = "6 semitones". One would expect the combination "2 + 4 + 5" to be correct here, but in reality this combination produces a pitch which is much too sharp.

3) "9 semitones" does not follow from "1 + 3 + 4", because 2 + 3 + 5 = 10 semitones, and because "4 + 5" = "7 semitones". One would expect the combination "1 + 4 + 5" to be correct here, but in reality this combination produces a pitch which is again much too sharp.

4) "10 semitones" does not follow from "2 + 3 + 4 + 5", because 1 + 3 + 5 + 2 = 11 semitones. If, however, "2 + 3 + 4" is taken to result in "8 semitones", then this combination follows from "8 semitones", but it does not follow directly from "9 semitones" ("1 + 3 + 4".) One would expect that the combination "1 + 2 + 3 + 4" would be correct here, and that the use of the fifth valve would not be necessary, just as it was

not necessary to lower the pitch by 1 semitone from "8 semitones". In reality, the combination "1 + 2 + 3 + 4" produces a pitch which is much sharper than the desired one, and the fifth valve must be used instead of valve 1.

5) Finally, the combination "1 + 2 + 3 + 4 + 5" for "11 semitones" does not follow from anything. It does not follow from the first table, because 2 + 1 + 3 + 5 + 2 = 13 semitones, and it does not follow from the second table, because given that "9 semitones" follows from "1 + 3 + 4", and that "10 semitones" follows from "2 + 3 + 4 + 5", one would expect the proper combination for "11 semitones" to be "1 + 3 + 4 + 5". In reality, this combination produces a pitch which is almost a semitone too sharp, and the further addition of valve 2 is required.

The following conclusions may be made from these observations:

A) Numbers 2), 3), and 4) above are consistent with respect to each other, in that each deviates by one semitone from the result one would expect by addition. They are not consistent with respect to numbers 1) and 5), because 1) does not deviate from the expected result at all and 5) deviates from the expected result by two semitones.

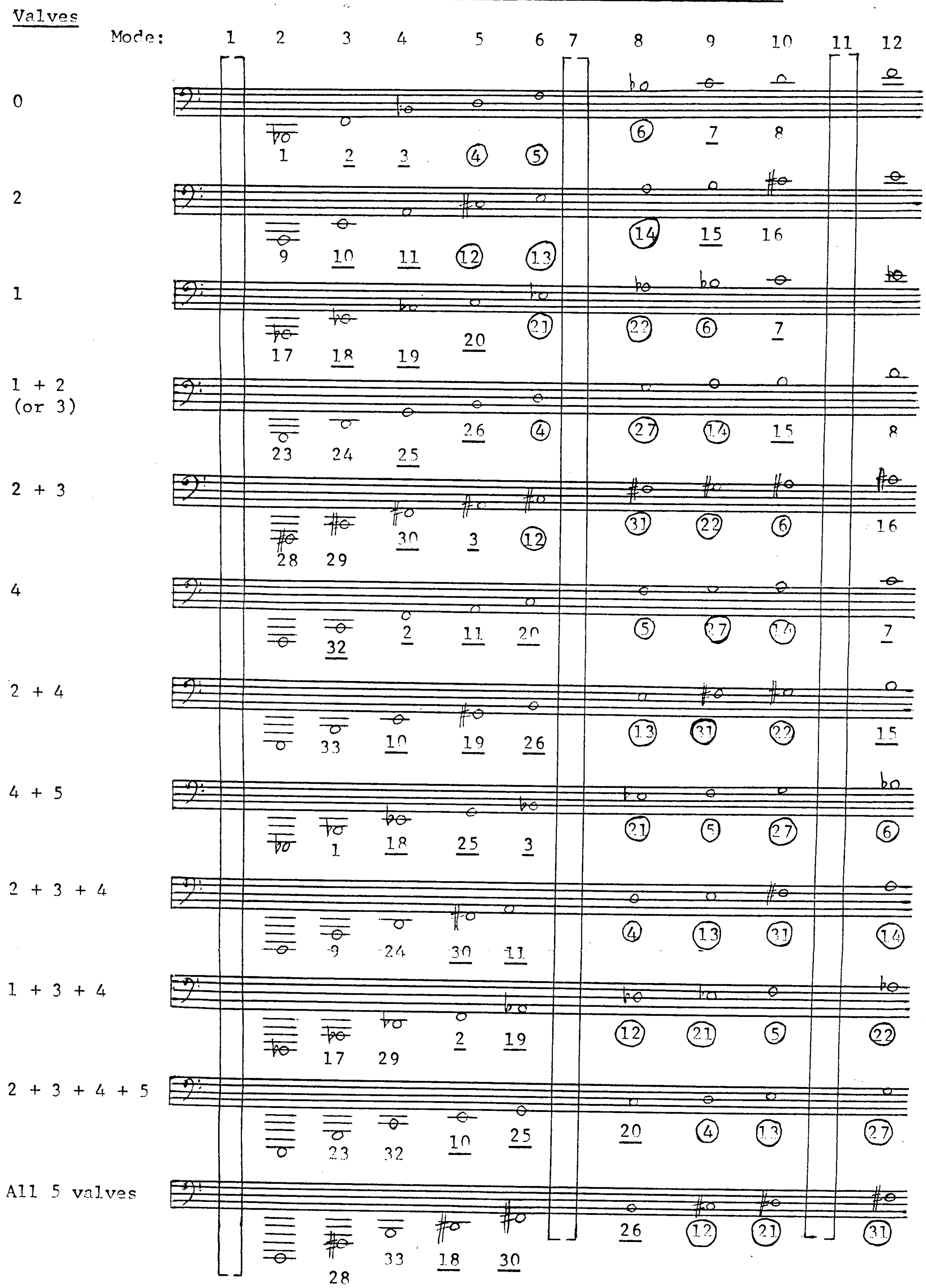
B) The major reason for all the discrepancies is the fundamental shortcoming (literally) of the valve system, especially in a range where great lengths of tubing are involved. Since the deficiencies of the valve system have already been discussed (on pages 131-136), all that remains to be added here is that the tuba provides the most vivid illustration of the problem because its length is the greatest of all the brass instruments.

C) The second reason for the discrepancies is the compromising principle upon which the fifth valve is designed. (In fact all valves are designed upon a compromising principle, but this shows up most clearly in the case of the fifth valve, because of the low range in which it operates.) This is best seen by comparing the fifth valve to valves 1 and 3. For "7 semitones" and "10 semitones" the fifth valve is just right; if valve 1 were used instead, the resulting pitch would be too sharp, and if either valves 1 + 2 or 3 were used, it would be too flat. For "8 semitones", the fifth valve is too sharp if it replaces valve 3. For "9 semitones", the fifth valve is too flat if it replaces valve 1 and too sharp if it replaces valve 3. Finally, as was already mentioned, for "11 semitones" it does not suffice to add the fifth valve to the valves used for "9 semitones" (1 + 3 + 4); valve 2 must also be added. All of this shows that the fifth valve is not designed so much to be a substitute for either valve 1 or valve 3, but to be an alternate for one of them, which sometimes yields better results, while at other times it yields worse results. Furthermore, it is incorrect to state categorically that the fifth valve lowers a pitch by any fixed number of semitones. On the contrary, its effect is different for each combination to which it belongs. Also, it must be remembered that slide adjustment during playing is what really makes this system workable, as was mentioned earlier.

For all these reasons, the valve combinations in the second table above do not follow any fixed pattern.

Through the use of the five valves in the manner indicated in the two tables above, the following pitches may (theoretically) be produced on the tuba:

Possible Written (and Sounding) Frequencies



This is so because each fundamental which results from a new combination of valves has its own overtone series, the members of which are accessible in each case by overblowing.

Although there are twelve rows containing nine notes each, only 43 of these 108 notes are different. These 43 notes have been subdivided into four categories as follows:

1) 10 pitches which appear only once -- unnumbered,

2) 11 pitches which appear twice each -- indicated with the same number in both places,

3) 12 pitches which appear three times each -- indicated with the same number underlined in all three places,

4) 10 pitches which appear four times each -- indicated with the same number circled in all four places.

The next step is to determine which valves the tuba player would, in practice, use to produce the 43 different pitches on his instrument. Clearly, for the 10 pitches for which only one fingering is possible there is no choice. These pitches are indicated in whole notes in the table on page 212. For the


Valves
Written (and Sounding) Frequencies Produced
Mode: 1 2 3 4 5 6 7 8 9 10 11 12
0
2
1
1 + 2 (or 3)
2 + 3
4
2 + 4
4 + 5
2 + 3 + 4
1 + 3 + 4
2 + 3 + 4 + 5
All 5 valves
8

remaining 33 pitches, the one is chosen which comes first (reading from top to bottom), for the same reasons as in the cases of the trumpet and french horn. In the table on page 212, these pitches are indicated as follows:

1) If the choice was made from two alternatives, the pitch is indicated with a half note,

2) If the choice was made from three alternatives, the pitch is indicated with a quarter note,

3) If the choice was made from four alternatives, the pitch is indicated with an eighth note.

These pitches are so designated in order to show that although the alternatives have not been indicated on the table, they do exist, and should not be totally ruled out. While the pitches which are indicated are first choices, the tuba player may wish to use some of the alternatives in at least two special situations:

1) in trilling or in fast passages, where technical ease may be greater with the use of an alternative,

2) in certain keys, where intonation of some of the notes may be better with the use of an alternative.

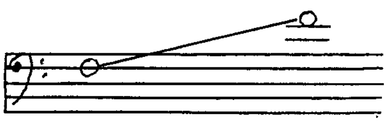
Consequently, it is to the tuba player's advantage to be aware of the alternate fingerings of the pitches of his instrument, and thus to have these at his disposal.

For two of the pitches (C#3 and D3) the second choices (in mode 6) are just as good, and often better. This is because the fifth mode is the "Achilles heel" on many tubas, and is better avoided if possible.[35] The two pitches have been indicated in bracketed eighth notes in the table.

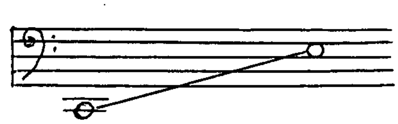
The table on page 212 clearly shows that the use of the fifth valve is necessary for only three of the 43 pitches of the normal range of the tuba. Expert players can extend the lower limit of this range considerably -- down to E0 (20.602 hz) -- a perfect fourth below the lowest note of the piano. (It follows that these players can produce pitches through the first mode of oscillation.) The extra notes which belong only to the exceptional range of the tuba are indicated in bracketed whole notes in the table on page 212.

In summary, then, the compass of the tuba is the following:

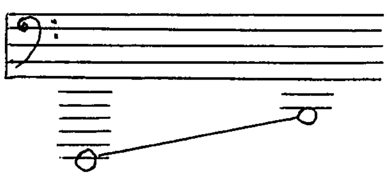
High Register:



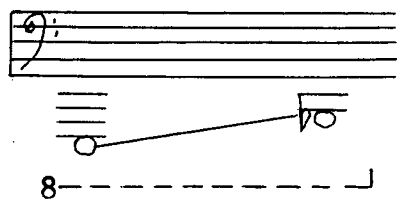
Middle Register:



Low Register:

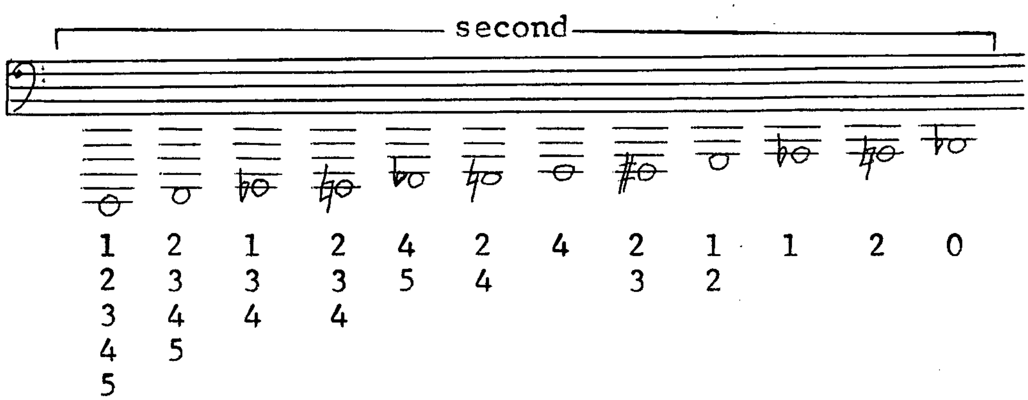


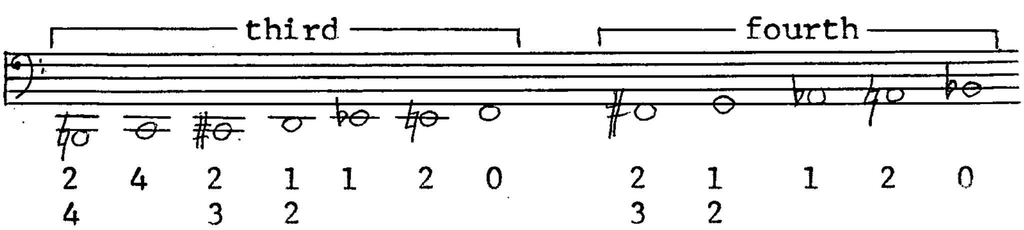
Extra-low Register
(exceptional):

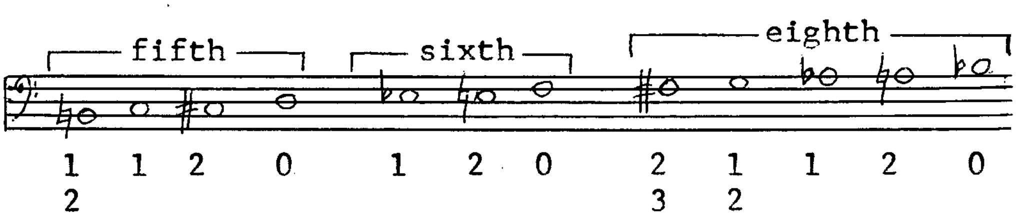


Naturally, the tuba is at its best in the middle register, where it is the easiest to play and its tone is the most characteristic. In the high register, the attacks are less sure and the extreme dynamic ranges (pp and ff) are more difficult to achieve. Similar problems are encountered in the low register, where the problem of air supply becomes quite serious, and speed of articulation is hampered. In the extra-low register, these additional problems are compounded, and a different embouchure is often used.

The complete chromatic scale of the normal range of the tuba may now be written out, showing the fingerings (by valve numbers) and the modes of oscillation used.[36]

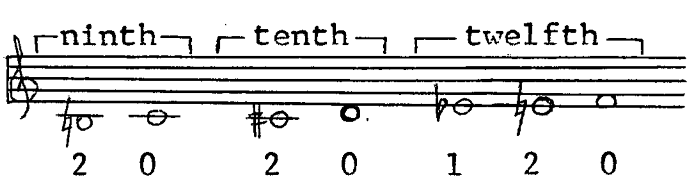






Alternate: 2 3 1 2

(sixth mode)



Testing the Equation $v = f\lambda$ [37]

As was done with the other brass instruments, we now wish to test the equation $v = f\lambda$ in a practical situation involving the tuba. Again, since the tuba is partly cylindrical and partly conical, neither $\lambda_{max} = 2L$ nor $\lambda_{max} = 4L$ holds true. Consequently, we must substitute known values of v and f into the equation, which will yield λ, which we will then compare to L.

To begin, we need to know the lengths of the parts of the tuba. These lengths were measured and found to be as follows:

Part	Length (in cm)
Mouthpiece	6.5
Leadpipe	63.0
Middlepipe	55.0
Bellpipe	354.0
Bell	76.0
Basic Length (equals total of above lengths)	554.5
Tubing added by valve 1	77
Tubing added by valve 2	38
Tubing added by valve 3	124
Tubing added by valve 4	207
Tubing added by valves 1 + 2	115
Tubing added by valves 2 + 3	162
Tubing added by valves 2 + 4	245
Tubing added by valves 2 + 3 + 4	369
Tubing added by valves 1 + 3 + 4	408

From these values we may easily find, by referring back to the fingering chart on page 212 and by addition, the total length of the pipe (L) used to produce seven of the pitches of the second mode of the tuba. We also need to include in the table the known frequencies of these pitches. Now, using v = 33853 cm/sec, we calculate λ for each of the seven pitches. Finally, we compare λ_{max} to L by solving for z the equation $\lambda_{max} = zL$ for each of the seven values of λ which were just calculated. The completed table, then, is found on the following page.

Ideal results would have been obtained if the value of z turned out identical for each examined pitch. The values obtained, however, range from .961 to .989 and average out to .973. Now it is more reasonable to assume that the average error in the calculations is .027 than to take the value of z to be precisely .973. Since this ratio is so close to 1.00, the "L+" factor does not need to be introduced. If we take λ_{max} to be equal to (1)L, we can test the equation $v = f\lambda$ by the same method that was used for the woodwind instruments. The results thus obtained are excellent, as shown in the table on page 220.

		F1	F#1	G1	Ab1	A1	Bb1	B1
1a	Examined Pitch:	F1	F#1	G1	Ab1	A1	Bb1	B1
1b	Known frequency of this pitch (hz):	43.654	46.249	48.999	51.913	55.000	58.270	61.735
	Length of pipe (L) (cm):	799.5	761.5	716.5	669.5	631.5	592.5	554.5
λ	(= 33853 cm/sec $\div$ f) (cm):	775.49	731.97	690.90	652.11	615.51	580.97	548.36
	z (= $\lambda_{max} \div$ L):	.970	.961	.964	.974	.975	.981	.989

1a Examined Pitch:	F1	F#1	G1	Ab1	A1	Bb1	B1
1b Known frequency of this pitch (hz):	43.654	46.249	48.999	51.913	55.000	58.270	61.735
Length of pipe (L)(cm):	799.5	761.5	716.5	669.5	631.5	592.5	554.5
λ (where λ_{max} = L)(cm):	799.5	761.5	716.5	669.5	631.5	592.5	554.5
Calculated frequency (33853 cm/sec $\div$ L)(hz):	42.343	44.455	47.248	50.565	53.607	57.136	61.051
2a Pitch whose frequency in equal temperament is nearest to that obtained above:	E1	F1	F#1	Ab1	A1	Bb1	B1
Known frequency of this pitch (hz):	41.203	43.654	46.249	51.913	55.000	58.270	61.735
Discrepancy of 2a with respect to 1a (by interval):	- 2nd lower	- 2nd lower	- 2nd lower	none	none	none	none
Ratio of calculated frequency with respect to 1b:	.973	.982	.979	.974	.975	.981	.989

V. PRACTICAL APPLICATION OF KNOWLEDGE OF THE ACOUSTICS OF WOODWIND AND BRASS INSTRUMENTS [1]

Having tested the equation $v = f\lambda$ and summarized the results for each instrument individually, the results will now be examined collectively and analyzed. Since the situation for the brass instruments is completely different from that of the woodwinds, the two families will be treated separately.

The table on the following page is a summary of the results which were obtained for each woodwind instrument. Column one does not consist of experimental results, but of formulas which were derived in Section II and used as premises in the testing of the equation. Regarding the discrepancies in column two, explanations for these were already given at the end of each respective section, therefore will not be repeated here. Column three represents more than just an alternate means of presenting the statistics given in column two. It is much more accurate than the former, simply because a semitone is too large a unit of measurement for these experiments (being subject to relative error of ±3% of the actual frequency in hertz.)

Instrument (see indicated page for complete table)	1) Known value of $\frac{\lambda max}{L}$	2) Average discrepancy of 2a with respect to 1a (in semitones)	3) Average ratio of calculated frequency with respect to 1b
Clarinet (p. 30)	4	$\frac{-3 + 3 + 2 + 0}{4} = .5$	1.04
Flute (p. 45)	2	$\frac{1 + 2 + 2 + 3}{4} = 2$	1.15
Oboe (p. 68)	2	$\frac{0 + 1 + 0 - 1}{4} = 0$	1.01
Bassoon (p. 94)	2	$\frac{2 + 1 + 1 - 2}{4} = .5$	1.03
Saxophone (p. 107)	2	$\frac{0 - 1 - 1 - 2}{4} = -1$	.955
		Overall average discrepancy: 0.4	Overall average ratio: 1.04

In summary, it should be emphasized that for the woodwind instruments these experiments were conducted solely out of curiosity -- to see how the values of f obtained by experiment would compare to those obtained by calculation.

With the brass instruments, the situation is so different, that strictly speaking, it is not correct to say that the equation $v = f\lambda$ was "tested", although this term was used to designate the corresponding section of the discussion of each instrument. If the equation were to be tested as it was for the woodwinds, the mathematics involved would be extremely complicated. Since in the case of the woodwinds the mathematics necessary for a first approximation could be done fairly simply, testing for validity of this first approximation could be carried out.

With the brass instruments, any discrepancies which result are not genuine discrepancies, because there is no known norm from which values can be observed to deviate. Furthermore, with instruments such as the tuba, where the bore diameter becomes extremely large, dimensions other than length have an effect on the behavior of sound waves within the tube; these were not taken into consideration.

That brass instruments do not behave in the manner that closed tubes are expected to behave is proven by the simple fact that all of them (even the trombone, which is approximately two-thirds cylindrical) produce both even and odd harmonics, although, being closed tubes, they should produce only odd harmonics. In this way, the construction of brass instruments can be considered similar to the construction of instruments employing conical tubes: alterations from the natural conical shape are made which result in capability of production of the same even and odd harmonics which are naturally produced by an open tube of equal length, as was explained in Section II.

The table on the following page is a summary of the results obtained for each brass instrument. Each column will be examined and analyzed individually.

Column One: Contrary to the case with the woodwind instruments, here the values of u, w, x, y, and z were not only unknown, but they were also complete-

Instrument (see indicated page for complete table)	1) Average calculated value of $\frac{\lambda max}{L}$ or $\frac{\lambda max}{L+}$	2) Average discrepancy of 2a with respect to 1a (in semitones)	3) Average ratio of calculated frequency with respect to 1b
Trumpet (pp. 140, 142)	Average of u = 1.01 = [.995 + 1.00 + 1.01(3) + 1.02 + 1.04] ÷ 7	$\frac{1 + 0(6)}{7} = .14$	1.01
French Horn; F side (pp. 171, 176)	Average of w = 1.05 = [1.04(2) + 1.05(2) + 1.06(2) + 1.08] ÷ 7	$\frac{0(6) + 1}{7} = .14$	1.01
French Horn; B-flat (pp. 172, 177)	Average of x = 1.05 = [1.04(3) + 1.05(3) + 1.07] ÷ 7	$\frac{0(6) + 1}{7} = .14$	1.01
Trombone (pp. 198, 200)	Average of y = [2.1(7)] ÷ 7 = 2.1	$\frac{0(7)}{7} = 0$	1.01
Tuba (pp. 219, 220)	Average of z = .973 $\frac{1}{.973} = 1.03$	$\frac{0(4) + 1(3)}{7} = .43$	.979
		Overall average discrepancy: .17	Overall average ratio: 1.00

ly unpredictable, because there was no basis upon which to make any predictions. At this point, it is easy to explain why the average value of y is the only one which is approximately equal to 2, while the others are approximately equal to 1: the trombone is the only brass instrument on which the complete first mode can be produced. If, for the other four instruments, there were an easy method of exciting the tube so that the fundamental pitches could be produced, then the first mode (rather than the second) would have been used for the calculations. (This could easily have been done in the case of the tuba.) Consequently, the values of λ would have been twice as large and the values of u, w, x, and z would have turned out to be 2.02, 2.1, 2.1, and 1.95 respectively. By comparing the table for the trombone to that of any of the other brass instruments, it can be clearly seen that: 1) this is a trivial case of working in a different octave, and 2) this has no significant bearing on the final results.

Upon observing that the discrepancy from the ideal result is at most 5% (in fact exactly 5% in the three cases where end correction was involved), there is basis upon which to make the following claim:

λ_{max} = 2L IS A FUNDAMENTAL MATHEMATICAL AND ACOUSTICAL PROPERTY OF BRASS INSTRUMENTS.

All discrepancies are attributable to exactly the same three factors which attributed to the discrepancies of f in the cases of the woodwind instruments (where the values of $\frac{\lambda_{max}}{L}$ are known.) These three factors are: 1) experimental error, 2) deviations from the "perfect" shape of the tube, and 3) the imperative simultaneous operation of <u>two</u> systems: the generating (or excitation) system and the resonating system, which was discussed on pages 69 - 71.

It must be clearly understood that the above claim is not the equivalent of claiming that λ_{max} = 2L is a property of an air column which is partly cylindrical and partly conical. Such a claim, if made, would have to be proven mathematically, and this proof would be a landmark in the field of acoustics.

The conclusion to which all of the above points is the following:

THE VITAL CHARACTERISTIC OF A BRASS INSTRUMENT IS NOT THE ACTUAL SHAPE OF ITS AIR COLUMN, BUT THAT THIS SHAPE -- WHATEVER IT BE -- POSSESS THE PROPERTY

$$\frac{\lambda_{max}}{L} = 2.$$

The theory to which the above claim and conclusion give birth is the following:

GIVEN ANY BRASS INSTRUMENT, THE NEARER THE VALUE OF $\frac{\lambda_{max}}{L}$ FOR THAT INSTRUMENT APPROACHES 2, THE MORE ACOUSTICALLY PERFECT IS THAT INSTRUMENT.

It must be added, that if an instrument were specifically designed so that its value for $\frac{\lambda_{max}}{L}$ equalled exactly 2, it may prove to have different, undesirable properties, but that would then be a separate problem. On the other hand, it is also possible that such an instrument would be acoustically superior in all other respects. It would certainly be worthwhile, therefore, to manufacture a brass instrument which would possess this property, with no regard for any of its other characteristics. The next step would be the elimination, one by one, of any undesirable properties which this instrument may have <u>without altering the value of</u> $\frac{\lambda_{max}}{L}$, i. e. <u>not</u> on a compromising principle. Such a task would be a formidable one indeed, and would probably take forever to achieve, but it could result in a next-to-perfect instrument.

Column Two: Out of the seven pitches which were tested for each brass instrument, only one pitch deviated by a semitone for three of the instruments tested, and none deviated in the case of the trombone. In every case except those of the trumpet and tuba, this is directly attributable to the fact that "L+" rather than "L" was used in the calculations. Since it was not necessary to use "L+" for the trumpet and the tuba, we can speculate about why this was necessary for the french horn and the trombone. The french horn is the only instrument employing hand insertion into the bell, and the trombone is the only valveless instrument. This means that the manner of operation of these two instruments differs considerably from that of the standard principles of operation of the trumpet and the tuba. The most noteworthy point is, however, that no deviation is greater than one semitone, and that there is no deviation at all (in semitones) for the great majority of pitches.

<u>Column Three:</u> The remarkable (and least expected) result obtained here is that the overall average ratio is exactly 1.00. From this result and those in the first two columns, the following conclusion may be reached:

THE EQUATION $V = F\lambda$ CAN BE SHOWN TO HOLD FOR BRASS INSTRUMENTS IF AND ONLY IF THE PROPER RELATIONSHIP OF λ TO L IS FIRST ESTABLISHED.

The "if" part of this conclusion is proven by the results in the table. The "only if" part can easily be proven by substituting values for u, w, x, y, and z other than those which were actually used. This conclusion is the strongest evidence there is to substantiate the claim, conclusion, and theory presented earlier.

One more observation may be made by examining the instruments individually: the trombone turns out to be by far the most acoustically <u>consistent</u> instrument of all. It is the only instrument in which there is no deviation in the calculated value of $\frac{\lambda max}{L}$, and the only instrument in which there is no discrepancy in semitones of 2a with respect to 1a, not even for a single pitch. This leads to the following theory:

COMPLETE CONTROL OVER NOTHING MORE THAN TUBE LENGTH ALONE IS ENOUGH TO MAKE AN INSTRUMENT THAT IS ACOUSTICALLY NEXT-TO-PERFECT.

This theory is actually very similar to the theory presented above. The important difference is that it follows from different conclusions. The results obtained for the trombone also serve to justify the introduction of the concept of "L+", which is discussed in greater detail below.

At this point, every claim, conclusion, and theory which has been presented must be qualified by pointing out that the small amount of testing which has been done here is highly insufficient for generalizations or for claiming universal application of the statements made. The statements are based on nothing more and nothing less than the experimentation which was performed, and should be regarded in this light. A large amount of further testing is necessary to either strengthen or weaken the validity of these statements.

Final Summary

In order to tie together everything that has been done, it is necessary to examine in greater detail the variables v, L, f, λ_{max}, and $\frac{\lambda_{max}}{L}$ individually and with respect to each other.

To begin with v, we recall that the velocity of sound is approximately 33145 cm/sec at 25° C, and that it increases by 59 cm/sec for each degree C above 25° C. For both woodwinds and brasses, it was decided to use the velocity of sound at body temperature (37° C), which is why v was taken to be 33145 + 59(12) = 33853 cm/sec. There was no account made of the fact that at the end of the instrument, v could have been as low as the velocity of sound at room temperature (20° C), therefore 33145 - 59(5) = 32850 cm/sec, i. e. approximately 3% lower. Also, within the instrument v could have been anywhere between these two values. Since, however, v was treated as a constant for both woodwinds and brasses, the situation did not change when the second family of instruments was tested. What must be realized, however, is that in view of the above, all resultant discrepancies are almost certainly due, in part, to the fact that v is not, in reality, the constant 33853 cm/sec that it was taken to be.

Looking next at L, we recall that L was always taken to be the measured length of the instrument, therefore a constant, although a different constant in each case. For the woodwind instruments, where the relationship between L and λ_{max} is known, these two variables were assumed to be in direct proportion to each other (4:1 or 2:1.)

The greatest difficulties arise when we begin dealing with the remainder of the variables. From the conclusion given on page 230 it follows that in order to test the equation $v = f\lambda$, it is not sufficient to know the values of v and L; the relationship between the open air value of λ and L must also be known. Since this relationship <u>is</u> known for woodwind instruments, only one variable remained to be solved for: f. Testing the equation was, therefore, very straightforward; the results showed considerable deviation between calculated (i. e. predicted) values and those obtained by experiment.

Taking the same approach for the brass instruments, since the relationship between the open air value of λ and L is not known, we reached a dead end. For this reason, as mentioned before, it is not entirely correct to say that the equation $v = f\lambda$ was "tested". More accurately, this equation was <u>used</u> in an attempt to find the correct values of <u>two</u> unknowns: $\frac{\lambda max}{L}$ and λ. Since there were already two unknowns, it was necessary to treat f as a constant. This, however, was a reluctantly made, forced decision, because the experiments for the woodwinds showed just how great the discrepancy between the calculated values of f and the actual values of f can be. Consequently, we were forced by circumstances to provide a "standing invitation" for discrepancy.

At any rate, the next step was to find the value of $\frac{\lambda max}{L}$. To do this, the equation $v = f\lambda$ was, in essence, substituted by the equation $v = f(kL)$, and this equation was solved for each instrument instead. (k was replaced by u, w, x, y, and z respectively.) This resulted in a simple relationship between λ_{max} and uL and zL, but not between λ_{max} and wL, xL, and yL. For these three instruments, end correction entered the picture (5%, 4.5%, and 5% respectively.) Consequently, the original assumption that λ_{max} is directly proportional to L (λ_{max} = kL) was replaced by the assumption that λ_{max} = kL+, and thus the concept of "L+" was introduced. The "+" part of "L+", however, was nothing more than a calculated guess.

Since the equation $v = f\lambda$ was used (or misused), in this way, the resultant discrepancy of λ_{max} from the desired value of 2 (or 1) cannot be considered to be a discrepancy of λ_{max} alone, but of λ_{max}, v, and f (which was assumed to be fixed, although it really was not.) There is no precise way of determining exactly how much of the total discrepancy is attributable to discrepancy in f alone, but it is a reasonable assumption that this discrepancy is approximately equal to what it was with the woodwind instruments.

In the final analysis, one question remains: What was the justification for choosing those values for the "+" part of "L+" which were chosen? In conjunction with what was said about the trombone above, the results obtained for this instrument point to the conclusion that the choices for the "+" part of "L+"

were well made. A much stronger and far more significant justification, however, is provided by the results in column three. The choices which were made so that the values of w, x, and y would approach 2 (or 1) turned out to yield results in column three which were identical, and extremely close to 1.00. It is precisely this justification which is offered to substantiate the claim made on page 227.

One more argument may be presented in defence of the choices for the "+" part of "L+" which were made. Let us suppose that the relationship of λ_{max} to L was unknown for woodwinds, and use the same approach that was used for brasses. The resulting tables for each instrument are given on the following page. We see that the averages of the variables a, b, c, d, and e are as follows:

Average of a (clarinet)	4.28	Ideal value of a	4
Average of b (flute)	2.30	Ideal value of b	2
Average of c (oboe)	2.02	Ideal value of c	2
Average of d (bassoon)	2.12	Ideal value of d	2
Average of e (saxophone)	1.91	Ideal value of e	2
	12.63		12

Average discrepancy: 12.63 ÷ 12 = 1.0525.

Thus we see that if it would have been necessary to introduce the concept of "L+" for the woodwinds, the

Clarinet (original table on page 30)

Frequency (f) (hz):	146.83	174.61	311.13	415.3
λ (= $\frac{33853 \text{ cm/sec}}{f}$):	260.56	193.88	108.81	81.51
Length of pipe (L):	66.8	41.5	23.8	20.4
$\lambda_{max} \div L$ (= a):	3.90	4.67	4.57	4.00

Flute (original table on page 45)

Frequency (f) (hz):	261.63	293.66	349.23	554.37
λ (= $\frac{33853 \text{ cm/sec}}{f}$):	129.39	115.28	96.94	61.07
Length of pipe (L):	60.0	52.0	43.0	23.6
$\lambda_{max} \div L$ (= b):	2.16	2.22	2.25	2.59

Oboe (original table on page 68)

Frequency (f) (hz):	233.08	246.94	369.99	523.25
λ (= $\frac{33853 \text{ cm/sec}}{f}$):	145.24	137.09	91.50	64.70
Length of pipe (L):	70.6	65.1	46.1	33.8
$\lambda_{max} \div L$ (= c):	2.06	2.11	1.98	1.91

Bassoon (original table on page 94)

Frequency (f) (hz):	58.27	97.99	174.61	261.63
λ (= $\frac{33853 \text{ cm/sec}}{f}$):	647.66	345.47	193.88	129.39
Length of pipe (L):	262.2	165.7	90.1	73.6
$\lambda_{max} \div L$ (= d):	2.47	2.09	2.15	1.76

Saxophone (original table on page 107)

Frequency (f) (hz):	138.59	207.65	261.63	329.63
λ (= $\frac{33853 \text{ cm/sec}}{f}$):	244.27	163.03	129.40	102.70
Length of pipe (L):	122.2	84.9	68.5	56.0
$\lambda_{max} \div L$ (= e):	2.00	1.92	1.89	1.83

"+" part of "L+" which would have been chosen would have been, on the average, 5.25% of the basic length, which is strikingly close to the average for the brasses (4.83%.)

Finally, a word of explanation is in order regarding the question of accuracy of figures. The accuracy of the figures in this paper is acceptable from the physicist's point of view, but not from the musician's. An error of, for example, a semitone is a small error mathematically, (being in the order of $\pm$3%) but musically it is a greater error than one of a perfect octave, which, in turn, is a large mathematical error, being of a factor of 2. Thus, from a musician's standpoint, measurements which result in a pitch deviation of even a quarter-tone are not accurate enough to be useable for practical purposes. On the other hand, a physicist would be delighted with such great accuracy. This difference in practical interpretations must be kept in mind when dealing with the mathematics of music.

CONCLUDING REMARKS

In the previous section, a rather strong effort was made to support a claim which cannot be supported by a mathematical proof: that $\frac{\lambda_{max}}{L} = 2$ is a fundamental property of brass instruments. This claim was based largely on the observation that brass instruments, as they are constructed today, already seem to possess the property $\frac{\lambda_{max}}{L} \approx 2$ -- for whatever reasons. This suggests that manufacturers have derived through experiment and experience a result which a theorist would now like to arrive at and to prove correct by using formulas and equations.

But this exact situation represents a historical trend. In general, the regular order of incidents is that _first_ a professional in a practical art determines empirically that a particular model is the most perfect one available, _then_ (in some cases many years later) a theorist or a mathematician or a physicist appears on the scene to say to him: "_You_ are correct; _I_ can prove it!"

[To make an analogy from outside the field of acoustics -- the writer was once told that when scientists assigned themselves the task of determining the shape of a structure which was the most resistant to wind and to environmental conditions in general, they ultimately found that the structure which they arrived at through highly sophisticated methods was in the exact shape of a tree. "The Creator was correct; they can prove it!"]

All of this may be summed up by one basic question regarding construction and manufacturing: "How does the method of formulas and equations compare to that of the "trial and error + experiment and experience" method? The writer posed precisely this question to the following seven manufacturers of woodwind and brass instruments:

1) Larilee Woodwind Manufacturers,
1700 Edwardsburg Rd.,
Elkhart, Indiana 46514, USA

2) Selmer Brass and Woodwind Manufacturers,
Box 310,
Indiana 46514, USA

3) G. Leblanc Corporation,
7019 30th Ave.,
Kenosha, Wisconsin 53140, USA

4) Norlin Musical Instruments Ltd.,
51 Nantucket Boulevard,
Scarborough, Ontario, M1P 2N6,
CANADA

5) Fox Products Corporation,
South Whitley, Indiana 46787, USA

6) Renold Schilke,
Schilke Music Products Inc.,
529 S. Wabash Ave.,
Chicago, Illinois 60605, USA

7) Yamaha Nippon Gakki Co., Ltd.,
P.O. Box 1, Hamamatsu, 430,
JAPAN

The following are (excerpts from) the replies received from the first five manufacturers:

1) No reply

2) No reply

3) Mailed an article by Arthur H. Benade entitled: "The Physics of Brasses". The article was quite technical, and included discussions of such topics as resonance peaks of a trumpetlike instrument, geometry of horn flare, the trombone bell, two different impedance-measuring apparatuses, impedance patterns, regimes of oscillation, the tone color of a trumpet, and the extension of the range of the french horn by hand insertion.

4) "As a general principle it may be stated that all reputable manufacturers employ a combination of theory and "bench-test" prior to the production of a prototype. It in turn is field-tested until the final design is settled upon. This applies to everything from mousetraps to jetliners."

"...the application of an established formula does not mean that the design of the finished instrument is immutable and incapable of improvement, and this is where the manufacturer's experience and ingenuity come in, to produce an instrument one or more steps better than his competitors - perhaps freer-blowing, better intonation, and what-have-you.

(signed)
W. C. Duncan,
Manager,
Customer Service"

5) "...the bore dimensions and tone hole sizes of all our instruments are determined by achieving an understanding of of the behavior of those sections and shaping them to achieve the desired result. There is no math or theoretical formulas involved and to my knowledge, which includes frequent contact with A. H. Benade at Case Institute in Cleveland, there are none which sufficiently deal with the problem to achieve the results that we are seeking.
You may find some more practical theoretical approaches if you explore the brass instrument area, which is somewhat simpler than the woodwind area,...

(signed)
Alan H. Fox,
President"

This last statement is a very interesting opinion indeed.

6) "In brass instruments as well as woodwind instruments, it is impossible to figure out your nodal patterns mathematically and the best way I found to find your pressure points and points of rarefaction is by the use of the con-

tact mike utilizing a high speed camera and an oscilloscope to pick up the pattern itself. In this way you can obtain exact patterns and it is the only way that it is possible. I have worked on the theory of utilizing the speed of sound and dividing it by the number of vibrations per second and worked on this theory for several years and came up with the complete fallacy as the accuracy of the points of rarefaction in accordance with the pressure points and in no case was the rescinding note at all accurate.

I have worked with the top physicists in the world, including the most outstanding one of them all, Dr. Aebi from Bergdorf, Switzerland and I have found that this is the only approach that is at all accurate and can lead to furthering the success of whatever instrument you are working on in woodwinds or brass, or for that matter the stringed instruments. In fact, anything that produces sound can be analyzed in this manner.

Enclosed please find a copy of The Physics of Inner Brass, and while this is not going into it very deep it was a lecture I gave for educators and as a result I had to avoid going into too deep of technical terms.

The fact that you are working with cylindrical as well as tapered tubes, conical and none of them definite but all as french curves, it is definitely impossible to use a mathematical formula for determining the length of an instrument, but the method that I suggested to you of using contact microphones in the manner I suggested is the only accurate way of doing it, because the actual density of the metal as it changes will give you definitely different lengths of tubing for each instrument.

(signed) Renold Schilke"

The article enclosed was very general in scope and included discussions of such topics as pressure points, the K factor, materials of manufacture, plating of instruments, and mouthpieces.

The reply from the Yamaha Company in Japan, which is quoted below, was the most interesting. The two portions of the last two paragraphs which have been underlined below are particularly priceless -- they sum up the entire situation exquisitely.

7) "Basically, we also use "trial & error plus experiment & experience" method in designing or improve our instruments. This principle is employed through our production process by way of the strict inspection for quality control and routine feed-back system to our design, which gives us the opportunities to understand the causal relation between design and result.
We of course experience copying the most desirable instrument available for study purpose, however it is only a close resemblance and incorporating the result of our experience and findings, we can make such resemblance better than the original in many cases.
We can find some theory of acoustics of wind instruments as by Benade, Cremer, Coltman, Elder, Backus, and so forth, however most of them are just theory and do not do much for practical purpose. Many of such books even include rather wrong or false story. For instance, many books say that the pitch of a horn is decided by its total length of tubing, but in reality, the form or taper of

tubing influences the pitch of horn as well as harmonic complex.
As you will realize, the mensuration of actual horn is neither conical or exponential. We actually use more than one hundred data to specify the exact shape of tubing and it is nearly impossible to get resonant frequency analytically. Also, the boundary condition of wind instruments at the mouthpipe or lead pipe is neither "closed" nor "open". Brass instruments are in the "closed" side, however it is also subjective to performer or performance style.
In conclusion, we would say that only "trial and error" method can be used in actual designing of instruments and from this point, we can continuously seek for an evolution of design. Some scientific system or equipment such as "wave equation" and "computer machine" can be used as an assistance for analysis or working out some data, however they are just an equipment to help us save time and do never take initiative role in designing instruments.

(signed)
S. Hasegawa,
Manager,
Export Dept."

FOOTNOTES

I. DIFFERENTIAL EQUATIONS OF WAVE-MOTION

[1]Unless otherwise stated, all the information found in Section I is based on the following source:
F. C. Champion, University Physics, Part Four, Wave-Motion and Sound (London and Glasgow: Blackie and Son Limited, 1948), pp. 1-8.
It is presented in a simplified version as taught by Dr. Roger Howard of the Department of Physics, University of British Columbia.

[2]Jess Stein, editorial director, The Random House College Dictionary, (New York: Random House, Inc., 1975), p. 1488.

[3]$\frac{\partial y}{dx}$ is the designation for "partial derivative of y with respect to x". This represents a ratio of small changes of y and x under the condition that all other factors remain constant.
$\frac{\partial^2 y}{dx^2}$ is the designation for "second partial derivative of y with respect to x". These concepts are introduced and explained in calculus textbooks.

II. THE CYLINDRICAL TUBE

Boundary Conditions for the Cylindrical Tube

[1]This subsection (to page 20) is based entirely on Section I. It consists of application of the raw material presented in Section I. All the equations and graphs presented on these pages are the generous contribution of Dr. Roger Howard of the Department of Physics, University of British Columbia.

Possible Frequencies

The Clarinet

[2]This result was obtained on page 16.

[3]Most of the discussion of the clarinet up to this point is based on the following source:
John Backus, The Acoustical Foundations of Music, (New York: W. W. Norton and Company Inc., 1969), p. 191.

[4]The fingering chart is quoted from:
Keith Stein, The Art of Clarinet Playing, (Evanston, Illinois: Summy-Birchard Company, 1958), p. 24.
The writer also consulted clarinettists, who demonstrated alternate fingerings for certain pitches -- especially those in the extreme register.

[5]The writer makes this inference from the subjective discussion found in:
Geoffrey Randall, The Clarinet, (London: Ernest Benn Limited, 1960), p. 37.

[6]See footnote 5 above.

[7]The idea of testing this equation in pracitcal situations involving the woodwind instruments is that of Dr. Roger Howard of the Department of Physics, University of British Columbia. No additional sources were necessary to carry out this experiment. This

applies to every experiment involving a woodwind instrument. The instruments which were used were chosen at random in every case.

[8]The velocity of sound may be considered common knowledge of physics.

[9]This entire subsection (to page 37) consists of a paraphrase of the discussion found in:
John Backus, Ibid., pp. 75, 203-206.

The Flute

[10]This result was obtained on pages 18-19.

[11]John Backus, Ibid., p. 188.

[12]The discussion of fingering is based on the following source:
Edwin Putnik, The Art of Flute Playing, (Evanston, Illinois: Summy-Birchard Company, 1970), p. 27.
The writer also consulted flute players, who demonstrated alternate fingerings for certain pitches.

[13]This diagram is adapted from:
John Backus, Ibid., p. 187.

[14]See footnote 7.

[15]John Backus, Ibid., p. 188.

III. THE CONICAL TUBE

[1]The first part of this section (to page 59) is a continuation of pages 1-20. As was the case with the material presented on pages 14-20, all the equations and graphs presented on these pages are the generous contribution of Dr. Roger Howard of the Department of Physics, University of British Columbia.

Possible Frequencies

The Oboe

[2]This paragraph contains a summary of information obtained from two sources: oboists consulted by the writer and the following book:
Philip Bate, The Oboe: An Outline of its History, Development and Construction, (New York: W. W. Norton and Co. Inc., 1975), pp. 3-4.

[3]This result was obtained on page 59.

[4]The discussion of fingering is based primarily on demonstrations by oboists whom the writer consulted. The following source was also used:
Robert Sprenkle, The Art of Oboe Playing, (Evanston, Illinois: Summy-Birchard Company, 1961), pp. 36-37.

[5]Philip Bate, Ibid., pp. 184-185.

[6]See footnote 7 under Section II.

[7]This entire subsection (to page 75) is a loose paraphrase of the following source:
Philip Bate, Ibid., pp. 123-129, 150-151.

The Bassoon

[8]The discussion of the bassoon up to this point is based on the following source:
John Backus, Ibid., pp. 195-196.

[9]The last two paragraphs are a paraphrase of an excerpt from the following source:
Lyndesay G. Langwill, The Bassoon and Contrabassoon, (New York: W. W. Norton and Co., Inc., 1975), p. 145.

[10]This result was obtained on page 59.

[11]The discussion of fingering is based primarily on demonstrations by bassoonists whom the writer consulted. The following source was also used:
Julius Weissenborn, Practical Method for the Bassoon, (New York: Carl Fischer Inc., 1958), Supplement.

[12]John Backus, Ibid., pp. 196-197.

[13]The last four paragraphs consist of a loose paraphrase of an excerpt from the following source:
Lyndesay G. Langwill, Ibid., pp. 146-148.

[14]John Backus, Ibid., P. 206.

[15]Philip Bate, Ibid., pp. 141-142.

[16]John Backus, Ibid., p. 88.

[17]See footnote 7 under Section II.

The Saxophone

[18]John Backus, Ibid., p. 197.

[19]This result was obtained on page 59.

[20]The discussion of fingering is based primarily on demonstrations by saxophonists whom the writer consulted. The following source was also used:
Larry Teal, The Art of Saxophone Playing, (Evanston, Illinois: Summy-Birchard Company, 1963), pp. 68-69.

[21]See footnote 7 under Section II.

IV. THE BRASS INSTRUMENTS

[1]This discussion (to page 115) consists largely of a loose paraphrase of an excerpt from the following source:
John Backus, Ibid., pp. 215-219.

[2]With the exception of one paragraph, this subsection (to page 117) is based on the following source: James H. Winter, The Brass Instruments, (Boston: Allyn and Bacon Inc., 1964), pp. 11-12.

[3]John Backus, Ibid., pp. 219-220.

[4]The discussion of the bell up to this point is based on the following source:
John Backus, Ibid., pp. 220-222.

[5]Philip Bate, The Trumpet and Trombone, (London: Ernest Benn Ltd., 1972), pp. 25-26.

[6]Ibid., p. 29.

[7]Ibid., pp. 15-16.

The Trumpet

[8]John Backus, Ibid., p. 221.

[9]Ibid., p. 223.

[10]Ibid., pp. 224-225.

[11]The format used in indicating pitches for the brass instruments is adapted from:
Paul Hindemith, The Craft of Musical Composition: Book I: Theoretical Part, (New York: Associated Music Publishers, Inc., 1937), p. 20.

[12]This table of fingerings was prepared in collaboration with performers. The same holds true for the corresponding tables for the other brass instruments.

[13]Arthur H. Benade, Horns, Strings, and Harmony, (New York, Doubleday and Co. Inc., 1960), p. 181.

[14]The discussion of the valves up to this point is based to a large extent on the following source:
John Backus, Ibid., pp. 225-226.

[15]The preceding three paragraphs consist largely of a paraphrase of an excerpt from the following source:
Robin Gregory, The Horn, (New York: Praeger Publishers, 1969), pp. 68-69.

[16]John Backus, Ibid., p. 227.

[17]A detailed discussion of the entire situation and of the problems involved in "testing" this equation for the brass instruments will be presented in Section V.

[18]Robin Gregory, Ibid., p. 22.

[19]The principal source of reference for this subsection up to this point was:
John Backus, Ibid., pp. 228-231.

[20]The principal source of reference for the preceding two pages was:

Robin Gregory, Ibid., pp. 23-24.
Performers were consulted by the writer for verification of details.

[21]This fingering chart represents the culminating point of the step-by-step derivation and determination of pitches and fingerings which began on page 148. For verification, the writer consulted performers, as well as the following two sources:
Robin Gregory, Ibid., pp. 70, 75,
William C. Robinson, An Illustrated Method for French Horn Playing, (Bloomington, Indiana: Wind Music Inc., 1968), pp. 64-65.

[22]Robin Gregory, Ibid., p. 50.

[23]Ibid., pp. 60-61.

[24]John Backus, Ibid., p. 231.

[25]See footnote 17.

The Trombone

[26]Philip Bate, The Trumpet and Trombone, (London: Ernest Benn Ltd., 1972), p. 47.

[27]John Backus, Ibid., p. 224.

[28]This chart represents the culminating point of the step-by-step derivation and determination of pitches and positions which began on page 181. For verification, the writer consulted performers, as well as the following two sources:
Philip Bate, The Trumpet and Trombone, (London: Ernest Benn, Ltd., 1972), p. 202.
Robin Gregory, The Trombone (New York: Praeger Publishers Inc., 1973), p. 63.

[29]John Backus, Ibid., pp. 223-224.

[30]Kent Kennan: *The Technique of Orchestration*, (New York: Prentice-Hall, Inc., 1952), p. 138.

[31]Robin Gregory, *The Trombone*, (New York: Praeger Publishers Inc., 1973), p. 86.

[32]Ibid., pp. 88-89.

[33]Adopted from Kent Kennan, Ibid., p. 138.

[34]See footnote 17.

The Tuba

[35]This information was obtained upon consultation with performers.

[36]This chart represents the culminating point of the step-by-step derivation and determination of pitches and positions which began on page 209. For verification, the writer consulted performers, as well as the following source:
Donald Wesley Stauffer, *A Treatise on the Tuba*, (Rochester, New York: University of Rochester Press, 1942), p. 64.

[37]See footnote 17.

V. PRACTICAL APPLICATIONS OF KNOWLEDGE OF THE ACOUSTICS OF WOODWIND AND BRASS INSTRUMENTS

[1]This section was prepared with the generous aid of Dr. Roger Howard of the Department of Physics, University of British Columbia.

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APPENDIX

Frequencies of Pitches in the Equal-tempered Scale.

CO	16.352	C3	130.81	C6	1046.5
C#O	17.324	C#3	138.59	C#6	1108.7
DO	18.354	D3	146.83	D6	1174.7
D#O	19.445	D#3	155.56	D#6	1244.5
EO	20.602	E3	164.81	E6	1318.5
FO	21.827	F3	174.61	F6	1396.9
F#O	23.125	F#3	185.00	F#6	1480.0
GO	24.500	G3	196.00	G6	1568.0
G#O	25.957	G#3	207.65	G#6	1661.2
AO	27.500*	A3	220.00	A6	1760.0
A#O	29.135	A#3	233.08	A#6	1864.7
BO	30.868	B3	246.94	B6	1975.5
C1	32.703	C4	261.63**	C7	2093.0
C#1	34.648	C#4	277.18	C#7	2217.5
D1	36.708	D4	293.66	D7	2349.3
D#1	38.891	D#4	311.13	D#7	2489.0
E1	41.203	E4	329.63	E7	2637.0
F1	43.654	F4	349.23	F7	2793.8
F#1	46.249	F#4	369.99	F#7	2960.0
G1	48.999	G4	392.00	G7	3136.0
G#1	51.913	G#4	415.30	G#7	3322.4
A1	55.000	A4	440.00	A7	3520.0
A#1	58.270	A#4	466.16	A#7	3729.3
B1	61.735	B4	493.88	B7	3951.1
C2	65.406	C5	523.25	***C8	4186.0
C#2	69.296	C#5	554.37	C#8	4434.9
D2	73.416	D5	587.33	D8	4698.6
D#2	77.782	D#5	622.25	D#8	4978.0
E2	82.407	E5	659.26	E8	5274.0
F2	87.307	F5	698.46	F8	5587.7
F#2	92.499	F#5	739.99	F#8	5919.9
G2	97.999	G5	783.99	G8	6271.9
G#2	103.83	G#5	830.61	G#8	6644.9
A2	110.00	A5	880.00	A8	7040.0
A#2	116.54	A#5	932.33	A#8	7458.6
B2	123.47	B5	987.77	B8	7902.1

*Lowest note of 88-key piano
**Middle C
***Highest note of 88-key piano